## TinkerP|ots Workshop Guide



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## Introduction

As the title suggests, this guide is meant to be used in a workshop setting, where participants can work together and a workshop leader can give demonstrations, lead discussions, and answer questions. To learn to use TinkerPlots ${ }^{\text {TM }}$ Dynamic Data Exploration software outside a workshop setting, consult the Walkthrough Guide that comes with the Instructor's Evaluation Edition or supplement the activities in this guide with the five introductory movies that come with TinkerPlots (choose TinkerPlots Movies from the Help menu).

## To the Workshop Participant

Welcome to TinkerPlots and the world of Dynamic Data. The activities in this guide, which begin on page 4 , will introduce you to the most important features of TinkerPlots and show you how you can use them to help your students understand data.

At the end of the workshop, take this booklet with you to explore and evaluate TinkerPlots in more detail. Remember that the TinkerPlots Workshop Guide is for evaluation purposes only and is not intended for classroom or other extended use. As you become comfortable using TinkerPlots, we encourage you to help others by leading workshops yourself.

## To the Presenter

We have found that a successful and simple workshop format for TinkerPlots involves little more than turning participants loose to work in pairs with the activities in this guide. (If you prefer that participants work individually, have them identify partners they can consult when problems arise.) The Presenter Notes suggest ways to introduce the activities to participants as well as places to stop and have discussions. It is extremely useful to have a computer that can project to a screen or large monitor so that everyone can see what you do. Remember that teachers are not used to sitting at a computer all day. Be sure to allow breaks for stretching between sessions. When participants encounter challenges, encourage self-reliance by suggesting that they read the margin notes, use the Quick Reference Card and TinkerPlots Help, and look through the menus. Show them how to scroll over a tool to see what it does. Encourage them to try things-they can always choose Edit | Undo to get back to where they were.
Key Curriculum Press grants copy permission to any presenter who wants to give this TinkerPlots Workshop Guide to workshop participants. Key Curriculum Press also grants permission to install multiple copies of the TinkerPlots Instructor's Evaluation Edition or a single network server copy for the purposes of presenting a workshop.

Preparing for the Workshop: In addition to previewing the activities and practicing the demonstrations, you should complete the following checklist before the workshop.

- Is the most recent version of TinkerPlots installed? To make sure you have the most recent maintenance update, look on the TinkerPlots Online Resource Center. (Choose Check for Updates from the Help menu.)
- Do TinkerPlots Help and Movies work properly? Because TinkerPlots Help opens in your web browser, this is worth checking. Does Help launch from the Help menu? Do the movies play, and is there sound? (Choose TinkerPlots Movies from the Help menu.)
- Are the computers properly connected to the Internet? If not, you'll need to work around this as you discuss importing data into TinkerPlots (page 12). You can have participants use the file Excel Import.xls, posted with the TinkerPlots Workshop Guide on the TinkerPlots Resource Center. Load this file onto participants' computers before the workshop starts.
- Are the necessary TinkerPlots files installed on participants' computers, and are they easily accessible? Participants will work mostly with the documents in the Data and Demos folder, which comes with TinkerPlots. Additionally, participants will need the TinkerPlots documents Hall of Fame.tp, Mammal Problems.tp, Scooters.tp, and US States Update.tp (this document corrects some errors in the original US States data set). If participants are using the Instructor's Evaluation Edition, they will also need the document Numbers.tp, as this is not included in the Sample Data and Demos. If participants are not connected to the Internet, they will also need the file Excel Import.xls, as explained above. These files are posted with the TinkerPlots Workshop Guide on the TinkerPlots Resource Center. (Choose Online Resource Center from the Help menu.) You'll need to load these files onto participants' computers before the workshop starts.
- Am I on the same platform as the participants? Be aware that if you are on a Macintosh and your participants are on Windows, or the other way around, there will be some slight differences in where the commands are located and in the way TinkerPlots looks. These differences are noted in the guide.
Coaching and Modeling: As you coach participants on teaching with TinkerPlots and TinkerPlots techniques, model behaviors you use with students. Be explicit about your modeling and encourage participants to use similar behaviors in their own classes. Use the end-of-session discussions to have participants describe and comment on the modeling they observed. Here are some behaviors to model (there's no need to model every behavior in each session):
- Introduce the learning and pedagogical goals at the start of each activity.
- Briefly describe the important techniques required by an activity.
- Do a quick run-through of building a useful plot before participants begin an activity.
- Interrupt mid-activity to demonstrate methods or describe pitfalls.
- Give participants plenty of time to respond to questions and to participate in discussions.
- Have participants summarize what they learned and what challenges they encountered.

This guide is meant to provide material for a one-day workshop. If you have less time, here are two suggested formats:
1 hour: Session 1, Beginning to Look at Data
3 hours: the above plus selections from Session 2, Gathering Data, and Session 3, Investigating Data

## About the Instructor's Evaluation Edition

If you do not have a TinkerPlots license, you may be using the Instructor's Evaluation Edition. This edition expires 60 days after first use. It has all the features of the full version, with a few important exceptions. You can open documents, import or enter data, do investigations, and use all the features of the full version, but you cannot save any documents you create, export the data you generate, copy pictures of TinkerPlots objects to paste into another application, or print your work.
The full version allows you to do all these things, and also comes with additional resources:

- Full set of Data and Demos—about 40 data sets and 9 demonstration files
- Exploring Data with TinkerPlots, which includes seven classroom-ready activities in blackline-master form


## Session 1 Beginning to Look at Data

## LEARNING GOALS FOR THIS SESSION

Make predictions about the relationship between two characteristics, make graphs to investigate a question, and support a conclusion using information from a graph.

## PEDAGOGICAL REFLECTION QUESTIONS

How does the investigation approach used in these activities engage students with the data (if it does)? What have students learned at the end of these activities?

## ACTIVITIES

- Introduction to TinkerPlots (whole-class presentation)
- Backpacks
- U.S. States


## RESPONSE TO PEDAGOGICAL REFLECTION QUESTIONS

## TO THE PRESENTER

This session will probably take about an hour. Times given are approximate.
5-10 min: Introduce TinkerPlots either by using the demonstration Introduction to TinkerPlots or by showing the movie TinkerPlots Basics. If you choose to do the demonstration, you might want to view the movie yourself while you are preparing, because the demonstration is based on the movie.

20 min: Participants start using TinkerPlots by answering a question about backpacks. Make sure you do not let this activity go on too long. Participants will get more practice with TinkerPlots in the next activity.

30-40 min: Participants complete the U.S. States activity. This activity often is very engaging for participants and is a good segue into a discussion about why we study data.

Explain that this demonstration can also be used to introduce students to TinkerPlots, or that TinkerPlots comes with a short QuickTime movie of the demonstration.

You may also want to make a case table and show how selecting a case in one representation highlights it in all representations.


The cats will be listed in order by their ages.

In this demonstration you'll use a pre-made file to show the structure of TinkerPlots and some basic plot features. Make sure participants don't try to follow alongthey'll have a chance to do this very soon. Take about 5 minutes. Explain each step of what you are doing.

1. Open TinkerPlots. Start with a blank document, expanded to fill your screen.
2. Choose File|Open, and go to the Data and Demos folder in the TinkerPlots folder. Open the Science and Nature folder and then the document Cats.tp. Explain that you can enter your own data into TinkerPlots but that it also comes with about 40 sample data sets. Most of these have data in different subject areas, and some show more advanced capabilities of TinkerPlots.
3. Explain that this document contains data about 24 cats. Each cat is represented by an icon in the plot. The data for each cat are shown in a data card. The attributes are described below the data card. Click on a few points in the plot and point out how this brings up their values.
4. Select the attribute Gender in the data cards, and then click the Key button to get a color key. Explain that we can color the icons in the plot according to their values for an attribute. Notice that now the males are yellow and the females are purple.
5. Select a numerical attribute such as Age. Explain that for a numerical attribute you get a color gradient-the older a cat is, the darker pink the icon is. Click on one of the older cats to show its values.
6. Right now the cases are randomly arranged in the plot. Click the Mix-up button to send them to a new random arrangement.
7. Explain that TinkerPlots does not have a menu of graph types. Instead, students build plots by progressively using basic operations that stack, order, and separate case icons. This process helps them understand how plots are constructed.
8. Select Age in the data cards if it isn't already selected.
9. Click the Order Horizontal button and explain that it orders the cases, with the younger cats on the left and the older cats on the right. Click it again to turn order off.
10. Click the Order Vertical button, explain what it does, and then turn it off.
11. Click the Stack buttons one at a time and explain what they do.
12. With the icons stacked vertically, select Name in the data cards and click the Label button.
13. Select Age in the data cards and click the Order Vertical button.

You could instead click the Separate Horizontal button.

If participants ask about the cat image icons, explain that you can attach an image icon to a document using the file Case Icon Library.tp in the Data and Demos folder.
14. Click the Mix-up button, and then click the Key button.
15. Click on one of the icons. Drag it to the right and let go. This separates the plot into two groups: young cats and old cats. Drag a little more until you have four or five groups. Click the Stack Vertical button.
16. Choose Circle Icon from the Icon Type menu, and then choose Fuse Rectangular. Explain that you can use icons other than cats to represent cases. Here each case is represented first by a circle and then by a rectangle.
17. Drag an icon to the right until the icons become fully separated, and each icon sits above its value on the horizontal axis. Then choose Circle Icon from the Icon Type menu and make the icons smaller.
18. To look at a different attribute, drag BodyLength from the data cards and drop it below the horizontal axis.


In these steps, you are modeling an inquiry, but it isn't necessary to say so explicitly.
19. Explain that you can use color in TinkerPlots to explore relationships between attributes. Point out that in this plot there are two clumps. Wonder aloud whether we have some younger cats that are shorter and some older cats that are longer.
20. Click on Age in the data cards if it isn't still selected. If we did have older cats that were longer, we'd expect to see more dark pink at the right and more light pink at the left. But we don't. Maybe the males are longer than the females.
21. Click on Gender in the data cards. The clump at the left is mostly females and the clump at the right mostly males, so it seems that we might be right.
22. To explore this idea further, drag an icon up. When you let go, the plot should separate, showing one graph for males and one for females.
23. Explain that you can use the toolbars to add features to your plot. Show the mean and reference lines by clicking the Mean and Ref. buttons. Use the Drawing tool to draw the shape of one of the distributions.
24. End the demonstration by choosing Edit | Undo to go back several steps. Explain that TinkerPlots has unlimited undo, so participants shouldn't be afraid to try something to see what happens.

In this activity participants start using TinkerPlots. There are no specific steps for them to follow. The purpose is for them to begin to be comfortable using TinkerPlots to make plots.

1. Explain that this activity follows the model of a five-part investigation: First participants will ask a question, then they will think about it (make hypotheses), then they will analyze the data, and finally they will use their analysis to support a conclusion. (The middle step of collecting the data has already been done for them.)
2. In this activity participants will use data provided with TinkerPlots to investigate the question: Do students in higher grades carry heavier backpacks? When students look at data, it's essential that they begin with a question and that they understand what the attributes mean. This example is a question dear to the hearts of many students, and the attributes are familiar to them.

Multiple copies of the Quick Reference Card come with full versions of TinkerPlots, and can help answer students' immediate questions.

If some participants finish early, have them investigate the question: Do males or females carry heavier backpacks?

The activity shows the formula editor as it appears on a Macintosh. The appearance is slightly different on Windows, but the instructions are exactly the same.
3. Show participants where to find the data set, and remind them to use the Quick Reference Card in the back of this guide if they have questions. Allow about 15 minutes for exploration. Don't prolong this activity.
4. After most people have a plot they are satisfied with, bring the group together for a brief discussion. Ask what their conclusions are: Do students in higher grades carry heavier backpacks? Ask volunteers to explain how their plots support their conclusion. Point out the very different plots that have been created around the room.
5. Bring out the need to look at body weight as well as backpack weight to decide the question, and demonstrate how to create a new attribute, PercentWt, and define it using a formula (the directions are in the activity).
Participants might bring up this need during the exploration. In that case, pause the session and do the demonstration earlier.
6. Have participants create the new attribute themselves and answer the last question in the activity. Briefly bring the group together to discuss their answers. This will probably take about 10 minutes.

## Backpacks

When students are looking at data, it's essential that they begin with a question and that they understand what each of the attributes means. In this activity, you'll look at data collected by students attempting to answer the question: Who carries the heaviest backpacks? You'll investigate part of that question.

1. Open TinkerPlots. Choose File |Open and go to the Data and Demos folder. Open the Exploring Data Starters folder and then the document Heaviest Backpacks.tp. This file contains data about the backpack weights of students in grades $1,3,5$, and 7 .
2. Look at each of the attributes. If you aren't sure what an attribute is, read the description below the data card.

## THINK ABOUT IT

You're now ready to explore the question: Do students in higher grades carry heavier backpacks?
3. Before you begin analyzing the data, think about what you expect to see. Briefly discuss the question with a partner.

## PLOT AND INVESTIGATE

4. Make two different plots that answer the question. Don't be afraid to try something to see what happens. Some tips:

- Refer to the Quick Reference Card at the end of this guide if you have a question.
- You can click on the buttons in the toolbars to add features to your plot.

C) Circle loon $\quad$ 园 $\leftarrow+|\theta|$.

- You can get a new plot by dragging the Plot icon from the toolbar into the document.
- If you do something you didn't mean to do, choose Edit |Undo. Sometimes you might get a plot that looks like this. If this happens, undo and think about what went wrong.

5. What is your conclusion for the question: Do students in higher grades carry heavier backpacks? How do your plots support your conclusion? Is this what you expected to see?

## Backpacks

## CREATE A NEW ATTRIBUTE USING A FORMULA

When you were looking at who carries the heaviest backpacks, it might have occurred to you that it would be good to consider the weight of the students as well as the weight of the backpacks. You can create a new attribute that will calculate the percent each student's backpack weight is of his or her body weight. Here's how.
7. In the data cards, double-click <new attribute> and type PercentWt.
8. Expand the data cards to the right until you see the Formula column. Doubleclick the circle for PercentWt.
9. Enter the formula into the formula editor. You can either type the attribute names or expand the Attributes list and double-click them.
10. When you're done, click OK. Each case should have a value for PercentWt. The values are gray to show that they were calculated.

11. Change your plots to look at PercentWt. Does this new information change your conclusion?

Tip: Use a reference line to find those students who are at risk.
12. If you finish early: Some doctors believe that students carrying backpacks that weigh more than $15 \%$ of their body weight might injure their backs. How many of these students are at risk?

To select multiple points, draw a selection rectangle around them.

See Exploring Data with TinkerPlots, Making and Researching Your Own Data, p. 17, for some ideas.
 select it in the data cards. Choose Edit | Delete Attribute. To hide an attribute, move it to the bottom of the list using cut and paste, and make the data cards smaller so it can't be seen.

The U.S. States activity helps participants get more enthusiastic about data analysis by using data they are probably more interested in than backpack weights. Participants also look at relationships between attributes in more detail. If your group includes more social studies teachers than math or science teachers, you might want to spend more time with this data set. In addition to doing the activity, have participants explore relationships on their own. Discuss what social studies concepts students could explore here.

1. Open the document US States.tp in the Data and Demos $\mid$ Social Studies folder and expand it to fill your screen. Explain that this data set was put together specifically for teachers rather than for students-it contains attributes teachers know and care about, such as NAEP scores and teacher salaries. Show participants how to expand the data cards to see all the attributes.
2. Point out that because we have the latitude and longitude of the capital cities, we have a rough map of the United States. We can use this map in several different ways to look at trends, or to pick out individual states. One way is to color the map with an attribute. For example, when you open the data set, the map is colored by Area. We can also use highlighting in two plots to look at trends.
3. Make a new plot, below the map, and turn off the attribute color by clicking on the blue bar by the word Attributes.
4. Make a line plot (dot plot) of Rainfall_Annual on the second plot: Drag Rainfall_Annual to the horizontal axis and fully separate.
5. Explain that you're going to highlight the states with low rainfall on the line plot. Ask participants where they think those states will be on the map. Then select the low-rainfall states. Were the participants right? Repeat with the highrainfall states.
6. Have participants complete the U.S. States activity. This will probably take about 30 minutes. Then lead the group in a discussion about the activity.

- What did you find in the U.S. States data set?
- What characteristics of this data set would make it challenging for middleschool students to use?
- What are some characteristics that you should look at when choosing data?
- What are some ways you could modify this data set to make it easier for students to use? (Show participants how to hide or delete attributes. Explain that they will work more on cleaning up data for student use in the next activity.)

In this activity you'll explore data from the 50 U.S. states. You'll look at how the states compare in terms of how much they spend on education, how they perform on NAEP, and whether there is evidence of a relationship between spending and educational performance.

## THINK ABOUT IT

Before you look at data, you should think about what you expect to see.
Q1 About how much do you think states typically spend per year per student?

Q2 List three states that you expect spend more than the average per student and three states that you expect spend less than the average.

More: $\qquad$
Less: $\qquad$
Q3 Estimate where you think your state ranks on each attribute.
My state

Public school spending
per student

| a. Ed_Spending: | Below average | Average | Above average |
| :--- | :--- | :--- | :--- |
| b. TeacherSalary: | Below average | Average | Above average |
| c. Gr8Math_NAEP: | Below average | Average | Above average |
| d. IncomePerCapita: | Below average | Average | Above average |

Q4 At the time these data were collected, 39 states had lotteries and 11 did not. Do you expect that states with lotteries spend more (per student) on average? If so, about how much more on average do you think they spend?

Q5 Do you expect to find a relationship between how much a state spends per student and the performance on math NAEP scores of eighth-graders?

## PLOT AND INVESTIGATE

Open the document US States.tp. For each question, make a graph that helps you answer the question. Consider how well your estimates reflect the data.

Q6 How much do states typically spend per year per student?
07 Make a graph that you can use to locate the six states you listed in Q2. Were you correct in your expectations?

Click the Div. and \% buttons on the upper plot toolbar.


See Binned Scatter Plot in TinkerPlots Help for details about how to make this graph.

08 For each attribute below, determine approximately what percent of states are ranked lower than your state.

This graph shows the percentile of Massachusetts for TeacherSalary (that is, what percent of states have lower teacher salaries than Massachusetts). The graph uses dividers and percents to show the percent of states that rank below Massachusetts.
a. Percentile for Ed_Spending
b. Percentile for TeacherSalary
c. Percentile for Gr8Math_NAEP
d. Percentile for IncomePerCapita

Q9 Do states with lotteries typically spend more per student? If so, about how much more do they typically spend?
$\mathbf{0 1 0}$ Is there a relationship between how much a state spends per student and the math NAEP scores of eighth-graders?

This graph is a good one to illustrate relationships between two numeric attributes. You can see the mean of TeacherSalary
 (blue triangles) increasing for higher levels of IncomePerCapita.

Q11 If you finish early, explore other attributes in the data set. Think about relationships that might exist between attributes and then make graphs to test your hypotheses.

## DISCUSSION QUESTIONS

1. Teachers have used this data set with sixth-grade students, and it has not been a pleasant experience! What about these data makes them particularly challenging for younger students?
2. What are some of the characteristics of data that you should consider when deciding whether to use a particular data set with your students?

## LEARNING GOALS FOR THIS SESSION

Be able to find or enter data and make it usable for your students.

## PEDAGOGICAL REFLECTION QUESTIONS

How do the data that students work with affect what they learn? How do you choose the data students will work with?

ACTIVITIES

- Importing Data into TinkerPlots
- Entering Data into TinkerPlots

RESPONSE TO PEDAGOGICAL REFLECTION QUESTIONS

## TO THE PRESENTER

This session probably will take about an hour. If you are short on time, omit the second part of Importing Data into TinkerPlots.

30-40 min: Demonstrate how to import data into TinkerPlots and have participants explore websites with data. They then practice fixing some problems common to data downloaded from the Internet.

20-30 min: Have participants work on entering data. The goal of this session is more for participants to understand what case-based data are and how to recognize them than for them to be able simply to enter data into TinkerPlots.

In this activity you'll first demonstrate how to download data from the Internet and then have participants download data for themselves and decide whether particular data sets are appropriate for their students. Both you and the participants need to be able to access the Internet. If you do not have Internet access, modify the first part of this activity by having participants import data from a Microsoft Excel file. See the notes below.

In either case, have participants watch while you demonstrate how to import the data into TinkerPlots. Make sure that participants don't try to follow along, perhaps by having them turn away from their computers.

Participants then use a pre-made file, Mammal Problems.tp, to practice finding and fixing common errors in imported data. You can skip this activity if you are short on time, but it will be valuable for participants when they try to work with their own data.

## IMPORTING DATA FROM EXCEL

1. Before the workshop, download the Excel document Excel Import.xls from the TinkerPlots Resource Center at www.keypress.com/tinkerplots/resources. Load this file onto all the workshop computers.
2. Explain that TinkerPlots can import data from many different applications, such as Microsoft Word, spreadsheet programs like Excel, and websites. There are three ways to import data: by copying and pasting, by importing from a text file, or by downloading from the Internet.
3. First we'll copy and paste. Open the Excel file. Highlight all the data, including the attribute names. Make sure nothing else is highlighted. Choose Copy from the Edit menu.
 into any TinkerPlots file, but using a blank document makes it easier for participants to follow.

Many programs can save documents as text files. Make sure the attribute names are in the first row and the document contains nothing but the data.
4. Open a new, blank TinkerPlots document. Make a new stack of data cards and make sure this stack is selected.
5. Choose Paste Cases from the Edit menu.
6. Make a graph or a case table to demonstrate that the data are there.
7. Optional: To demonstrate how to import from a text file, go back to Excel. Choose Save As from the File menu and save the data as a Text (Tab delimited) file. In TinkerPlots, make sure nothing is selected and then choose Import From File from the File menu. Select the file you want to import, and click Open.

## Importing Data into TinkerPlots

You'll learn more about the difference between summary data and case-based data in the next activity.

There is an amazing amount of data on the web. Many of the data you'll find online are summary data, and you can't do much with those when you download them into TinkerPlots. But when the data are case-based-that is, when they give you individual pieces of information (values of attributes) for many cases-you can download them into TinkerPlots and investigate them. In this activity you'll learn how to import data from the web into TinkerPlots and how to solve problems that can occur when you download data.

## DOWNLOADING DATA FROM THE WEB

The Data and Story Library (DASL) is an online library of data sets useful for education settings. All the data on DASL are case-based and are formatted so that they export into TinkerPlots with no problems.

1. Open a new TinkerPlots document, then open your web browser and position it to the right of your screen so that you can see the TinkerPlots document on the left.
2. Type DASL into a search engine to locate the Data and Story Library. In DASL, click on Data Subjects, and then select one of the several subjects offered there. Within the subject, click on a particular data file. At the top of the page, you can read a description of the data set. Below the description are the actual data Note that cases are listed in separate rows and that each column is a separate attribute.
3. To download the data into TinkerPlots quickly, drag the URL of the web page that contains the data and drop it into the blank TinkerPlots document. The data appear in a new stack of data cards.

Try following the links from the TinkerPlots Resource Center: www.keypress.com/ tinkerplots/resources.
4. If you want to add the comments or attribute descriptions to your TinkerPlots document, copy the text. Then in TinkerPlots make a new text box. Choose Paste Text from the Edit menu.
5. Investigate at least three data sets and list them below. Indicate whether you think each would be appropriate for your students, and give reasons for your decision. If you know of other sources on the web, look there.

## FIXING PROBLEMS IN DATA

Often when you download data from the web, it won't be as clean and tidy as the data from DASL. In this activity, you'll use a data set someone else has downloaded from the web to practice finding and fixing common problems.
6. Open the document Mammal Problems.tp.
7. Find as many errors as you can and fix them. You can do this on your own or by following the directions below. Some tips:

- Use a case table to look at all the data at once.
- If the attribute names weren't imported correctly, they might be the values of the first case.
- You can create new attributes using formulas to fix many problems. See steps 12-18 for an example.
- Units should go in the Unit column. They should not be part of attribute names or values.
- Remember that numerical attributes (those with numbers as values) should have a color gradient and that categorical attributes (those with words as values) should have a band of discrete colors.
- Blank values sometimes are represented with an asterisk (*). In TinkerPlots, blank values should actually be blank.

8. Make a case table by dragging a new table into the document.

One of the first things you'll probably notice is that the attributes have default names, such as ATTR1 and ATTR2.

You can press Tab to move to the next attribute name.
9. Rename the attributes by double-clicking an attribute name and typing in a new name. Use the values for case 1 to determine the proper attribute names.
10. Notice that height is in meters. Delete "meters" from the attribute name and type it in the Unit column.
11. The first case shows the attribute names rather than meaningful values. To delete that case, click the blue circle by the case number or click the row number in the table. Choose Edit |Delete Case.

If you don't clear the formula before you delete the original attribute, all your values for MassTemp will disappear. If this happens, choose Edit|Undo.

Asterisks (*) are sometimes used to represent blank values. In TinkerPlots, blank values should actually be blank.

Follow these steps to fix a problem by creating a new attribute using a formula. Here the values for the attribute Mass include the unit "kg." We want to delete the unit from all the values (without editing them one by one).
12. Make a new attribute called MassTemp.
13. Expand the data cards until you can see the Formula column. Double-click the circle for MassTemp.
14. Enter this formula into the formula editor and click OK. The text function stringToNumber looks for numbers in a value and deletes any text.


The values for MassTemp should match the values for Mass. The MassTemp values are gray to show that they were calculated.
15. Select MassTemp and choose Clear Formula from the Edit menu.
16. Select Mass and choose Delete Attribute from the Edit menu. Then rename MassTemp as Mass.
17. Type kg in the Unit field for Mass.
18. If you want to move the attribute, select it and choose Edit | Cut Attribute. Then select the attribute below where you want it to appear and choose Edit | Paste Attributes.

Remember that numeric attributes should have a color gradient and that categorical attributes should have a discrete color for each value.
19. Look for attributes with the wrong type of color bar. To see what's wrong, either graph these (drag the attribute to the vertical axis to see a list of values) or look in the case table. Fix any errors.

The values for Diet are "plants," "animals," and "both." Suppose you want students to practice using the terms "herbivore," "carnivore," and "omnivore." You can easily change the values by creating a new attribute using a switch-statement.
20. Create a new DietTemp attribute and double-click its formula circle to show the formula editor.
21. Enter the formula shown and click OK. This formula tells TinkerPlots to give DietTemp values based on the values of Diet but switched.

Tips on entering the formula:

- Once you type switch( all the parts of the statement will appear.
- Type words inside double quotes (a pair of double quotes appears when you type one).
- To get the middle line of the statement, position the cursor at the end of the first line and press Ins (Win) Option+Enter (Mac).


You can use switch-statements for many different things in TinkerPlots. If you want to change only two values, you can use something similar called an if-statement. See TinkerPlots Help: Working with Formulas for details.
22. Clear the formula for DietTemp and delete Diet as you did with MassTemp in steps 15 and 16.
23. If you finish early, explore the Mammals data set. Here's one question you might look at: Is there a relationship between how long mammals sleep and how long they live? If you think there is, describe it and try to explain it.

In this activity participants start to think about how to build a data set from scratch. What types of data work well in TinkerPlots? How do you structure data to allow students to explore them?

1. Introduce the tree activity simply by presenting the problem. Have participants work in pairs for 5-10 minutes to enter the data.
2. Ask for volunteers to explain what they did. Participants likely will have used two different methods: Some will have entered summary data, and some will have entered case-based data.

Summary data aggregate the information in some way. Here the totals are given for the number of trees of each type in each area. In TinkerPlots, a case is the information on a data card, or in a row of a case table, so here the case is not an individual tree, but rather a tree type.

These examples do not show the data that participants are using.

For more information about the importance of the case, see Research on Student Thinking About Data, page 60.


| Tree Data |  |  |  |  |  |  |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Tree_type | Area1 | Area2 | Area3 |  |  |  |  |  |  |
| $\mathbf{1}$ | Sugar Maple | 78 | 33 | 5 |  |  |  |  |  |  |
| $\mathbf{2}$ | Paw Paw | 11 | 22 | 0 |  |  |  |  |  |  |
| $\mathbf{3}$ | Witch Hazel | 4 | 17 | 0 |  |  |  |  |  |  |
| $\mathbf{4}$ | Hackberry | 1 | 0 | 19 |  |  |  |  |  |  |

Case-based data, or microdata, give data for each individual. Here each individual tree is entered separately. The case is a tree.

3. Discuss the advantages and disadvantages of each method. Bring out the distinction between summary data and case-based data. TinkerPlots is structured to work with case-based data. However, how you define a case depends on what you want to do with the data. For example, if we were comparing global properties of chinaberries to global properties of sugar maples, then it would make sense to use the tree type as the case. (This is how the mammals data were set up.) The subtlety of this distinction makes it important for teachers and students to understand what the case should be in any data-gathering situation. If they do not, they are likely to be surprised or confused when they graph their data. In the tree activity, for example, participants were probably expecting to see about 30 cases. Instead, they might see only four.
4. Have participants work in pairs on the next part of the activity. Here they need to build a data set from scratch. Take 5-10 minutes for this.
5. Discuss participants' reactions to this activity.

- What problems did you encounter, either in choosing a topic or in entering the data?
- The trees activity is based on a real data-gathering activity and on problems that arose during that activity. Can you think of other data-gathering situations in which you or students might be confused about what the case should be?


## Entering Data into TinkerPlots

For class projects, you might have to build a data set from scratch. To do this, open a new TinkerPlots document, drag a new stack of data cards into the document, and begin entering attribute names and values for each case. The information for each case goes on a separate data card. You can also make a case table and enter the data there.

## TREES

The diagram below shows four different species of trees growing near a river. There are two different areas: close to the river and far from the river. You need to enter data into TinkerPlots to answer this question: Does one type of tree grow more in one area than in another?


## COLLECTING DATA

A case-based data set is a collection of cases and their attributes. There might be a collection of individual

- students, where the attributes are age, gender, name, and height
- marbles, where the attributes are diameter, color, type, and manufacturer
- class episodes, where the attributes are number of minutes, current activity, number of students on task, and room temperature

Your task is to collect cases and enter the data into TinkerPlots. The collection should have these properties:

- It cannot be a collection of people-this is too easy!
- The collection should not be information you get off the web. It should be something you and your partner can observe in your immediate environment.
- It should include at least five cases and three attributes.
- The attributes should include at least one numeric and one categorical attribute.


## REFLECTION ON THE ACTIVITY

Summarize any difficulties you had, either in deciding on a collection or in entering the data appropriately into TinkerPlots.

## LEARNING GOALS FOR THIS SESSION

Investigate properties of whole numbers, understand how outliers affect means and medians, and understand the relationship between a scatter plot of two attributes and a line plot (dot plot) of the ratio of those two attributes.

## PEDAGOGICAL REFLECTION QUESTIONS

Does viewing numbers as data change your perception of them? How does using TinkerPlots make the concepts in these activities clearer (if it does)?

- Numbers (subsidiary activities Factors and Number Properties from Exploring Data with TinkerPlots)
- Changing Means and Medians (from Digging into Data with TinkerPlots by Amy Brodesky, Amy Doherty, and James Stoddard, Key Curriculum Press, 2008)
- Connecting Line Plots and Scatter Plots (from Digging into Data with TinkerPlots)


## RESPONSE TO PEDAGOGICAL REFLECTION QUESTIONS

## TO THE PRESENTER

In this session participants move from using TinkerPlots primarily as a tool for data analysis and exploration to looking at some ways they can use TinkerPlots to illustrate mathematical concepts. If your group includes more science teachers than math teachers, you'll want to focus more on the ways to use TinkerPlots to explore relationships than on the purely mathematical exploration of numbers. This session will probably take about 2 hours. You can demonstrate one or more of these activities if you're short on time.

30-45 min: Participants explore properties of the whole numbers from 1 to 100.
40-50 min: Model the activity Changing Means and Medians as a complete lesson.
30-40 min: Participants complete two worksheets on connecting line plots (dot plots) with scatter plots.

5-15 min: Optional: Show one or more of the three TinkerPlots movies on teaching techniques: Making Common Graphs, Comparing Groups, and Exploring Relationships. Watch these as a group or have participants watch them on their own. If participants don't have headphones, watching the movies on their own computers will be very loud and distracting because the sounds will not be synchronized.

This activity is a purely mathematical exploration. Participants use graphs that show numbers as points of data to look at patterns in number behavior.

1. Introduce the activity by opening the document Numbers.tp in the Data and Demos $\mid$ Science and Nature folder and demonstrating how to change the size of the stack.
2. Have participants spend about 20 minutes with this document. They can work on the Numbers activity, which was written for teachers, or on the Factors and Number Properties activities, which were written for students. Make sure participants see the activity notes following the activities. The Extensions contain other questions that can be investigated with this data set.
3. Lead participants in a discussion of what mathematics they could teach using this data set.

If you ever lose your stack, see steps 11-13 to get it back.

In this activity you'll use TinkerPlots to explore something we don't normally think of as data-numbers.

1. Choose File $\mid$ Open. Go to the Data and Demos folder, then the Science and Nature folder. Open the document Numbers.tp.

Here we have different properties for each number from 1 to 100 , such as whether the number is a multiple of various numbers, whether the number is a perfect square, and whether it's prime. The plot is already set up to be an ordered stack.
2. Change the size of the stack by dragging the bottom right corner of the plot. What kinds of questions could you ask students here?
3. Click on one of the factor attributes, such as factor8. Experiment with finding different patterns by changing the stack. What mathematical properties do these patterns show?
4. Click on a different factor attribute and compare the patterns. When does the pattern in one stack appear in another stack?
5. Click on the attribute prime. What's different about the patterns found in this stack? Can you make stacks that have roughly regular patterns?

You can also use different types of icons to look at patterns.
6. Choose Fuse Circular from the menu at the bottom of the plot. Here you can see that the primes aren't evenly distributed. Try clicking on some of the other attributes to look at their distributions. What do you notice?
7. Choose Square Icon from the menu to go back to a stack.

The data cards also contain attributes that show each number's digit sum and ones digit. You can use these attributes to look at many patterns as well.
8. Drag digit_sum from the data card to the horizontal axis of the plot.
9. Drag a plot icon to the right until there are no more bin lines. Then, using the slider on the plot, make the icons small enough that they don't overlap.


Why does the plot have this shape?
10. Click on factor3. What arithmetic rule does this show? What about factor 9 ?

These are just a few of the number patterns you can look at using this data set. Spend the next few minutes finding others. If you like, answer one of the questions in the document, or look at the Factors and Number Properties activities on pages 24 and 27 . Be sure to check out the activity notes on pages $30-36$ for some interesting extensions. These activities are from the book Exploring Data with TinkerPlots that comes with TinkerPlots.

Tip: To get a stack, follow these steps:
11. Drag the Plot icon from the toolbar into the document to make a new plot, or click the Mix-up button on the old plot.
12. Click on number in the data cards and then choose Square Icon from the menu in the plot. If labels aren't showing, click the Label button.
13. Click the vertical Order button, then the vertical Stack button.

## Factors

$\qquad$
In this activity you will explore patterns related to multiplication. You'll look at properties of the numbers 1 to 100. At right are the data for the number 24 . Notice that the value for factor 3 is "yes." This means that 3 is a factor of 24 . You can see that $2,3,4,6,8$, and 12 are all factors of 24 .

## Plot and Investigate

Now you'll look at the data to see what patterns you can find.

1. Open the document Factors.tp. You'll see a stack of data cards like the one at right.
2. You'll also see a plot of square icons stacked and ordered by number. Drag the right and bottom edges of the plot. You'll see the square icons move around to form
 stacks of different sizes.
3. Drag the edges or corner of the plot so that the stack is 3 wide, as shown here. You can also use the Icon Size slider to make the squares fit better.

Q1 Describe any patterns you notice in this stack.

|  | 28 | 29 | 30 |
| :---: | :---: | :---: | :---: |
| E 24 | 25 | 26 | 27 |
| E | 22 | 23 | 24 |
| ลิ 18 | 19 | 20 | 21 |
| İd | 16 | 17 | 18 |
| - 12 | 13 | 14 | 15 |
|  | 10 | 11 | 12 |
| E 6 | 7 | 8 | 9 |
| O | 4 | 5 | 6 |
| 0 | 1 | 2 | 3 |
| (2) | n |  | - |

4. Now select the attribute factor 3 in the data cards. The color of the plot should change. (The key above the plot tells you what the colors mean.)

Q2 Describe any patterns you notice in the stack now.

Q3 What do you think will happen if you make the stack 5 wide and select the attribute factor5? Write your prediction, and then try it.

Q4 In general, what will happen if you make a stack of a particular width and select the same factor as the width? Why does this happen? Include a copy of at least one graph that backs up your answer.

You probably know that division is the inverse (or opposite) of multiplication. You can use your plot to explore division too.
5. Think about the division problem $24 \div 8$. You probably already know the answer is 3 . This means that there are 3 groups of 8 in 24 (or 8 groups of 3 in 24 ). Because you are dividing by 8 , make a stack 8 wide and select factor8. Look for 24 in the stack.

Q5 What does the color of the icon for 24 tell you?

Q6 How high is 24 in the stack? How does this relate to $24 \div 8$ ?

## Factors (continued)

6. Think about the division problem $24 \div 8$ again. Because you are dividing 24 , make the stack 24 wide but keep factor 8 selected. Look for 24 .

Q7 How does this plot show you that $24 \div 8=3$ ?

Q8 Use either method of making a plot to do these division problems. For at least one problem, include a copy of your plot and explain how it gives you the answer.
a. $18 \div 3=$
b. $26 \div 13=$
c. $11 \div 2=$
d. $8 \div 3=$
7. As you change the size of the stack, you probably see the factors create different patterns in the stack. Experiment with different sizes and different factors to find one pattern that is interesting to you. Include a copy of this plot with your assignment.

Q9 Describe why this pattern is interesting to you. If you can, explain how the width and factor combine to create the pattern.
$\qquad$
In this activity you will explore the whole numbers 1 to 100. You'll look at three types of whole numbers.
Square numbers are produced by multiplying a number by itself, such as $4 \times 4=16$. You say that 16 is a perfect square. You can represent a perfect square number by arranging objects in the shape of a square. At right is a square made up of 16 smaller squares, showing that 16 is a perfect square.


Triangular numbers can be represented by objects arranged in the shape of an equilateral triangle. Here are triangles that represent the first four triangular numbers. What are the numbers?



Prime numbers have only two factors- 1 and the number itself. For example, 5 is a prime number because its only factors are 1 and 5.
At right are the data for the number 36. The "yes" and "no" values for the attributes show that 36 is a perfect square and a triangular number, but it is not prime.

## Plot and Investigate

Now you'll look at the data to see what patterns you can find. Let's start with the
 perfect squares.

1. Open the document Number Properties.tp. You'll see a stack of data cards like the one above.
2. You'll also see a plot of square icons ordered by number. Drag the bottom edge, right edge, or bottom-right corner to make a stack that is 4 wide, as shown at right. You can also use the Icon Size slider to make the squares fit better, but don't make them too small.
3. Select the attribute perfect_square in the data
 cards. The color of the plot will change, as shown at right. Include a copy of this plot with your assignment.

## Number Properties (continued)

Q1 Do you see a pattern to the square numbers in the plot? Describe the pattern as fully as you can.

Q2 Assume that you have data for numbers greater than 100. Based on the pattern that you see, where will the next five perfect squares appear? If you can, figure out the value of those square numbers.
4. Drag the edges or corner of the plot to change the size of the stack. Find another stack that helps you see a pattern to the square numbers. Include a copy of this plot with your assignment.

Q3 Fully describe the pattern that you see in this stack. Explain how this pattern compares with the pattern that you saw in Q1.

Now you'll look at the triangular numbers.
5. Select the attribute triangular in the data cards. Change your plot or make a new plot that shows a pattern to the triangular numbers. Feel free to experiment with different options. For example, you don't have to use square icons. Include a copy of your plot with your assignment.

Q4 Describe the pattern that you see in the triangular numbers as completely as you can. Based on your pattern, what are the next five triangular numbers greater than 100 ?

## Number Properties (continued)

Now you'll explore the prime numbers.
6. Select the attribute prime in the data cards.
7. Make at least one plot that shows possible patterns for the prime numbers. Include a copy of your plot(s) with your assignment.

Q5 Describe any possible patterns that you see in the prime numbers. Your answer should say how your plots help you see these patterns. Can you use the patterns to predict the next five prime numbers greater than 100 ?
minimum or maximum body weight or for using a range of values rather than one absolute value.) You might extend this into a discussion about how teachers, parents, and school board members might use this information when adopting textbooks or shaping school policy.
4. This activity focused on differences between grade levels. Ask students to continue analyzing the data to find differences between genders or body weights. Body weight is particularly interesting because students can debate to what degree body weight depends on grade level. (This raises the distinction between correlation and causation. Does gaining body weight cause students to carry heavier backpacks? Probably not. Rather, as students get older, their body weight increases, they move into higher grades, and they have more homework.)
For students in grades 6-8, you can even begin to look at bivariate relationships between attributes. For example, what does a scatter plot of BodyWeight and PackWeight tell you? (As body weight increases, the backpack weight tends to increase, but there are plenty of exceptions.) If students are novices with scatter plots, other types of plots, such as binned scatter plots, may be an easier way for them to recognize the relationship between the two attributes.

## Links

You and your students can find links to additional data and information about backpack weight and safety at the TinkerPlots Online Resource Center. Choose Online Resource Center from the Help menu or go to www.keypress.com/tinkerplots.

## Factors (page 24)

Activity Time: 30-55 minutes
Required Document: Factors.tp from the folder Data and Demos | Exploring Data Starters. This document uses the same data as Number Properties.tp in this folder and Numbers.tp in the folder Data and Demos | Science and Nature. In each document the attributes are ordered by their use in the activity.

Prerequisites: Students need to know basic multiplication and division of whole numbers. Preferably, students are familiar with the mathematical term factor.

Mathematics: Students will recognize different representations for the same number; select, apply, and translate among mathematical representations; describe numbers in terms of their factors; understand various meanings of multiplication and division; and understand the inverse relationship between multiplication and division.
Connections: This activity is primarily mathematical. However, students will enjoy the patterns that result from changing the width of the plot and coloring by different factors. This can lead them to general observations that they need not formalize. For example, to get perfectly diagonal patterns, you make the stack one wider or narrower than the factor.
General Notes: This activity explores the connection between multiplication and division, and between factors and multiples. Students use roughly rectangular stacks of the numbers 1 to 100 and color every multiple of a specific factor. The patterns created by the colored numbers help them identify every multiple of the factor.
The primary goal of this activity is for students to explore how they can describe whole numbers in terms of their factors. Most middle-school students have probably had experience representing multiplication with a rectangular arrangement of blocks. This activity takes advantage of that familiarity and builds on it by allowing students to quickly change the height and width of the rectangle to see what products result.
The student worksheet is relatively straightforward, giving step-by-step instructions for creating a few plots and asking specific questions about those plots. For students in grades 6-8, you may want to break away from the worksheet and encourage students to do independent explorations of the factor patterns. Students will find them compelling
and describe many patterns. The extensions at the end of these activity notes offer suggestions for going even further into algebraic concepts and primes.
A secondary goal is to show how you can use TinkerPlots to explore non-statistical concepts and data. This activity in particular uses data about the characteristics of numbers to develop number sense and explore multiple representations.

## Plot and Investigate

Q1 Answers will vary depending on grade level.
If students have left the icons colored by number, they may notice only that the color gets darker as the numbers increase.
Older students may notice patterns as they go up the columns, such as " 3 is added each time I go up." They may also notice that the third column represents the multiples of 3 .
Q2 Students should identify the third column as "multiples of 3." Some students may appropriately use the terminology defined at the beginning of the worksheet and say that these are the numbers with " 3 as a factor."


Q3 Predictions will vary. An ideal answer is to guess that a stack 5 wide and colored by factor 5 will have all the multiples of 5 , or the numbers with 5 as a factor, in the last column.
Although this question may seem obvious to you, and to many students, it is a necessary building block for some students. The goal of this question is to allow students who don't see the connection between factors and multiples an opportunity to use inductive reasoning - to recognize a pattern and make a generalization. Allowing time to explore the pattern will strengthen students' reasoning skills and help them make the connection between factors and multiples. In fact, you may prefer to have students repeat $\mathbf{Q 3}$ for a few other width-factor combinations before moving on to Q4.
Q4 In general, a stack $X$ wide and colored by factor $X$ will have all multiples of $X$, or all numbers with $X$ as a factor, in the last column.

Note: Obviously, many students will not be able to state this in these general terms, but they may be able to give several numeric examples.
After trying several width-factor combinations, students should be able to explain why this happens: "The numbers with 5 as a factor are all multiples of 5 . So, if the stack is 5 wide, all the multiples of 5 will be on the end of the rows."
Q5 The icon for 24 will be colored like all of the numbers with 8 as a factor. This means that 8 is a factor of 24 , and 8 divides into 24 with no remainder.

Q6 The icon for 24 is in the 3rd row of the stack. The 3rd row is related to the answer 3.


You may want to pause here and help students see the connection to familiar rectangular diagrams for multiplication. If you look at the rectangle that is defined by 24 , it is 8 wide and 3 high, or $8 \times 3=24$. It is because of the inverse relationship between multiplication and division that we are able to use this rectangular model to solve $24 \div 8=3$.
Q7 This plot shows that there are 3 complete groups of 8 in 24 and this another way of explaining why $24 \div 8=3$.

This solution represents division by what may be another familiar process - making equal groups. However, because the row of icons is continuous, some students may have trouble recognizing it as groups.
If you like, you can point out one way to translate between the representations in $\mathbf{Q 6}$ and Q7: take the rectangle in $\mathbf{Q 6}$; imagine cutting the rows into 3 separate rows; and then put the rows end to end.
Q8 Students can use either method for these division problems. The answers below show alternating methods. The divisions with remainders will be more challenging, although not impossible. Whether students answer with fractions or remainders will depend on grade level and proficiency with division and fractions.
a. 6 ; a stack 3 wide and colored by factor 3 shows 18 is in the 6th row.
b. 2; a stack 26 wide and colored by factor 13 shows 13 goes in 2 times.
c. $5 \frac{1}{2}$, or 5 remainder 1 ; a stack 2 wide and colored by factor 2 shows 11 is 1 more than the 5 th row, or $5 \frac{1}{2}$ rows.
d. $2 \frac{2}{3}$, or 2 remainder 2 ; a stack 8 wide and colored by factor 3 shows that 3 goes in 2 whole times with 2 out of 3 left over.
Q9 Answers will vary.

## Extensions

1. As written, this activity always uses a plot of stacked, ordered, square icons. Challenge students to experiment with other types of plots that show interesting patterns. For example, this ordered fused circle colored by factor 10 shows how multiples of 10 are evenly spaced throughout the numbers from 1 to 100.

2. Have students create two plots stacked and ordered by number and look for relationships between them. They can try two stacks of different widths colored by the same factor, or two stacks of the same width colored by different factors. Challenge students to explain why the relationships exist. For example, these two plots show that every multiple of 6 is also a multiple of 3 , but only half of the multiples of 3 are multiples of 6 . This is easily explained because 3 is itself a factor of 6 . Students could even compare a third plot-factor 2 - and see that every multiple of 6 is also a multiple of 2 , but only a third of the multiples of 2 are multiples of 6 .

3. Students can begin to incorporate some of the other attributes such as even, digit_sum, or ones_digit. Exploring how these relate to the factor attributes can introduce or reinforce many division "rules," such as "all even numbers are divisible by 2 " and "if a number ends in 0 or 5 , it is divisible by 5 ." The plot below illustrates the rule "if the sum of the digits is a multiple of 9 , then the number is divisible by $9 .{ }^{\prime \prime}$

4. Students in grades $6-8$ who are beginning to learn about prime numbers can use the stacks and factor attributes to explore the concept of prime. Because a prime number only has factors of 1 and itself, the prime numbers will never appear in the last column of a stack, except for the first row. As an example, consider 17. If you make stacks $2,3,4, \ldots, 15$, 16 , and 17 wide, the only time 17 will be in the last column is when the stack is 17 wide. So the only factors of 17 are 1 and 17 , and 17 is prime.
5. Students in prealgebra or algebra could write algebraic expressions for the patterns of multiples in the stack. Consider this stack that is 4 wide and colored by factor 17 :


The first multiple of 17 is 1 more than the 4th row of 4 , the second multiple of 17 is 2 more than the 8 th row of 4 , and so on. Students could write the sequence of numbers as

$$
\begin{aligned}
& 4 \times 4+1=17 \\
& 4 \times 8+2=34 \\
& 4 \times 12+3=51 \\
& 4 \times 16+4=68
\end{aligned}
$$

Advanced students could even generalize the sequence for the $n$th multiple of 17:

$$
4 \times 4 n+n=17 n
$$

## Number Properties (page 27)

Activity Time: 55+ minutes
Required Document: Number Properties.tp from the folder Data and Demos \| Exploring Data
Starters. This document uses the same data as Factors.tp in this folder and Numbers.tp in the folder Data and Demos | Science and Nature. In each document the attributes are ordered by their use in the activity.

Prerequisites: Students are familiar with square numbers, prime numbers, and triangular numbers. These types of numbers are described on the worksheet, but having prior experience with these classes of numbers will be extremely helpful.
It is best if students have first done Factors and worked with the Numbers data. That activity will give students experience with the type of data contained in the TinkerPlots document and with making stacks of rectangular icons.
Mathematics: Students will select, apply, and translate among mathematical representations; describe classes of counting numbers according to characteristics (specifically, square numbers, triangular numbers, and prime numbers); represent, describe, extend, and generalize geometric and numeric patterns.
Connections: This activity is primarily mathematical. However, students will enjoy the patterns that result from changing the width of the plot and coloring by different factors. This can lead them to general observations that they need not formalize. For example, to get perfectly diagonal patterns, you make the stack one wider or narrower than the factor.
General Notes: The primary goal of this activity is for students to recognize and generalize patterns in types of counting numbers. Specifically, students will create different arrangements of the numbers from 1 to 100; color all of the square numbers, triangular numbers, or prime numbers; and then describe, explain, and extend the patterns they see.
A secondary goal is to show how you can use TinkerPlots to explore non-statistical concepts and data. This activity, in particular, uses data about classes of numbers to develop number sense and pattern recognition.
Because students will be doing open-ended exploration of perfect squares and primes, it is best that they are familiar with these classes of numbers. Although the student worksheet begins with brief
descriptions, this activity is not intended as a comprehensive introduction of the concepts. Therefore, the activity is best suited for students in grades 6-8.

This activity is rather long and may take more than one class period to complete. Because the activity is divided into three explorations-squares, triangular numbers, and primes-you could break it up over three consecutive half-days. You could also divide the activity into three completely separate activities that you do independent of each other, or you could pick and choose from among them.

A final note about vocabulary: The worksheet uses square number and perfect square synonymously. This could confuse some students. Also, the collection includes attributes called square and perfect_square. For this activity, students need to use perfect_square, which tells whether the number is a perfect square; square is a numeric attribute that gives the value when you square the number.

## Plot and Investigate

Q1 Answers will vary depending on grade level and pattern-recognition skills.
In the simplest case, students should notice that the perfect squares are always in the first or fourth column. They might further notice that the perfect squares in the first column are always odd and that those in the fourth column are always even.
Some students may notice patterns in the number of cases between the perfect squares. For example, in the first column the second perfect square is 2 rows above the first, the third perfect square is 4 rows above the second, the fourth perfect square is 6 rows above the third, and so on. Furthermore, in the fourth column the second perfect square is 3 rows above the first, the third perfect square is 5 rows above the second, the fourth perfect square is 7 rows above the third, and so on. (Note: Some students may describe the separation by the number of rows between, but not including, the perfect squares, in which case they might describe the first column's pattern as " 1 row between . . . , 3 rows between . . . , etc.")

Some students may notice that consecutive perfect squares increase by odds: " 4 is 3 more than 1,9 is 5 more than 4,16 is 7 more than $9, \ldots$. . However, because this insight requires reading across the rows rather than focusing on the columns of highlighted numbers, it may be a rare response.

Note: Some students might prefer to swap the axes in order to make the plot fit the window better. This is okay, but then the patterns in the numbers go from left to right.


Q2 The first perfect square after 100 will be in the first column, 10 rows above 81 . The second will be in the fourth column, 11 rows above 100. The next will be in the first column, 12 rows higher. The next will be in the fourth column, 13 rows higher. And the fifth will be in the first column, 14 rows higher. (Note: Accept any answer that gets at this insight, including sketches that attempt to extend the rectangular stack.)
Some students may recognize that the numbers in each column increase by 4 as you go up, so they might calculate the perfect squares as

$$
\begin{aligned}
& 81+(4 \times 10)=121 \\
& 100+(4 \times 11)=144 \\
& 121+(4 \times 12)=169 \\
& 144+(4 \times 13)=196 \\
& 169+(4 \times 14)=225
\end{aligned}
$$

Other students might sketch an extension of the diagram and fill in or count spaces to find the numbers.
Be aware that some students may simply calculate $11 \times 11,12 \times 12$, and so on. Although this is an effective way to find consecutive perfect squares, it does not use the pattern illustrated by the plot.

Q3 Answers will vary, although there are two ideal answers.
First, a stack that is 2 wide shows a pattern of perfect squares alternating from side to side. If you look at the distance between perfect squares in each column, the pattern is similar to the stack that was 4 wide, except the evens and odds are not consecutive. For example in the first column, you go up 4 rows, then 8 rows, then 12 rows. Other students might notice a pattern based on alternating back and forth: "go up 1 row and right, go up 3 rows and left, go up 3 rows and right, go up 5 rows and left, go up 5 rows and right, ..."
Second, a stack that is 1 wide (or a continuous string) shows a pattern of increasing by odds: "up 3, up 5, up 7, up 9,..."

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Some students might start experimenting with types of plots other than stacks of rectangular icons. For example, a fused circular graph shows the increasing-by-odds pattern, too.
Q4 Plots and answers will vary. The next five triangular numbers are $15,21,28,36$, and 45 . A good answer is that triangular numbers increase by consecutive whole numbers: " go up 2, go up 3 , go up 4 , go up $5, \ldots$. . Both the single stack and fused circular graph below show this pattern. (Note: If students count the case icons between triangular numbers, the pattern starts at 1.) To help students see this pattern, you might have them go back to the geometric pattern on the worksheet and construct the next numbers in the sequence. They should see that you makethe next triangle by adding a bottom row one unit longer than the previous bottom row.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |



Also, triangular numbers always come in pairs of odds and evens: 1, 3 (two odds), 6, 10 (two evens), 15, 21 (two odds), 28, 36 (two evens), and so on. One way to highlight this pattern is to separate the icons by triangular and then color them by even. Again, you could ask students to go back to the triangular diagrams and challenge them to explain why this happens.


Q5 Plots and answers will vary. Unlike the perfect squares and triangular numbers, the primes have no consistent patterns. Some plots will
appear to show patterns, but these patterns can't be extended for all primes. There are some things students can say, however, about primes, such as the fact that other than the number 2 , they are all odd numbers.
Two interesting stacks of square icons are 5 wide and 6 wide, as shown below. The 5 -wide stack shows that the primes tend to fall on certain diagonals. The 6 -wide stack shows that the primes tend to be in the first and fifth rows. (Students could investigate why this is.)


From having used fused circular plots for triangular numbers, some students might do the same for the primes. They might notice a slight increase in the distance between primes as the numbers get closer to 100 .

## Extensions

1. Students will likely notice that there is an attribute called perfect_cube. You might define cubic numbers and have them look for patterns in the perfect cubes. Because there are only four perfect cubes less than 100, students might have a difficult time detecting any patterns.

You may want to have students add additional cases, say for 101 to 250 (or higher), in order to see more of a pattern. Note: Most of the attributes in Number Properties.tp are calculated with formulas, so it will be very easy for students to add more cases. (Choose New Cases from the Data menu and enter the desired number of new cases. Values for all the attributes except prime will be automatically computed.)
2. Students will also likely notice that there are attributes called square, square root, and cube. These numeric attributes perform the indicated function on number. For students in grades 6-8 who are beginning to learn about function plots, you can create graphs of these attributes by fully separating number on the horizontal axis and fully separating square, for example, on the vertical axis. Students can study the general shape of these function curves and describe similarities and differences.
3. You may be familiar with the Sieve of Eratosthenes. Eratosthenes (276-194 B.C.) was a Greek mathematician who devised a way to "strain out" prime numbers from composites. Essentially, because prime numbers cannot be a multiple of another number, you cross out all numbers that you know are multiples of $2,3,4$, 5 , and so on. You could do the Sieve with TinkerPlots, but it is better to do it with an oldfashioned number grid on a sheet of paper. Draw a number grid, or print a stacked plot, and cross off all the multiples of 2 , then all the multiples of 3 , and so on, up through the multiples of 17 . Students will notice that for several of the factors no additional numbers get crossed out - their factors have already been eliminated. For example, when you get to 10 you find that all its multiples have already been eliminated by its factors 2 and 5 .

## Links

You and your students can find links to information about square, triangular, and prime numbers at the TinkerPlots Online Resource Center. Choose Online Resource Center from the Help menu or go to www.keypress.com/tinkerplots.

This activity is taken from Digging into Data with TinkerPlots, Key Curriculum Press, 2008.

In this activity participants dynamically manipulate data to explore the effects of outliers on means and medians. This activity requires the document Scooters.tp, which you need to download from the TinkerPlots Online Resource Center and load onto participants' computers. Although you can have participants complete the activity on their own, we suggest that you use this activity to model how they could use TinkerPlots in the classroom by treating it as a genuine lesson.

1. Explain that in this activity you are going to model how you would use TinkerPlots in the classroom with your own students. Use the lesson notes on pages 41-46 to introduce the activity. Explain that this lesson follows one in which students looked at the ages of people who sustained scooter injuries. Make sure that all participants understand the definition of median.
2. Participants probably will spend about 20 minutes on the worksheet.
3. Use the questions at the end of the lesson plan to summarize the activity. Point out the lesson notes to participants, and discuss the teaching techniques involved in the lesson.

- How did using the software make the concepts clearer (if it did)?
- Were there any ways using the software got in the way of the concepts?


## How Do Outliers Affect Means and Medians?

Name:


When you look at the ages of the people who got scooter injuries, one person stands out from the rest: the 41 -year-old woman. This case is called an outlier because its value for age is unusually high in comparison to the other age values in the data set. (A case with an unusually low age value would also be an outlier.) In this lesson you'll explore how this outlier affects the mean and median ages for the data set.

1. Open the TinkerPlots file Scooters.tp. Make a line plot of the ages. Lock the scale by double-clicking the gray box at the right end of the axis and clicking OK.
2. Find the mean and median ages for people who got scooter injuries. Fill in the first row of the table below. (To find the values, click the arrow to the right of the mean button and choose Show Numeric Value(s).)

3. What do you think will happen to the mean and median if you remove the data for the 41 -year-old? Circle your choices.

The mean will: Stay the same Go up Go down
The median will: Stay the same Go up Go down
4. Hide the data for the 41-year-old: Click on the dot for the 41-year-old. Go to the Plot menu and choose Hide Selected
Cases. Fill in the rest of the table. Round to the nearest hundredth.

|  | Mean (yr) | Median (yr) |
| :--- | :--- | :--- |
| Data set with all $\mathbf{2 8}$ cases |  |  |
| Data set without the 41-year-old |  |  |
| How much did the measure change? |  |  |

5. Why did the mean change more than the median?

## How Do Outliers Affect Means and Medians?

continued
6. Put back the data for the 41-year-old: Go to the Plot menu and choose Show Hidden Cases. Then mix up the data and make a new line plot of Age.
7. What is the youngest age you think someone would be riding a scooter? $\qquad$
What is the oldest age you think someone would be riding a scooter?
$\qquad$
8. Change the scale to go from the youngest to the oldest ages you chose: Double-click the box at the left end of the scale. Enter the youngest age for Axis starts at and click OK. Repeat at the right end for the oldest age.
9. Change the age of the 41 -year-old case to see what happens to the mean and median. There are two ways to change the age.

- Click on the dot for the 41 -year-old to show her data card. Click on the value 41 and type in a new age.
- Choose the Drag Value tool. Click on the dot for the 41-year-old. Move the dot along the scale to see how the values for mean and median change. The age will change on the data card.

Keep your notes in the table (round to the nearest hundredth).

| New ages I tried (yr) | New mean (yr) | New median (yr) |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

10. What is the lowest mean and the lowest median you can get by changing this one age? (Do not use an age below the youngest age you chose in question 7.)
a. Lowest mean: $\qquad$ What age did you use? $\qquad$
b. Lowest median: $\qquad$ What age did you use? $\qquad$

## How Do Outliers Affect Means and Medians?

11. What is the highest mean and the highest median you can get by changing this one age? (Do not use an age above the oldest age you chose in question 7.)
a. Highest mean: $\qquad$ What age did you use? $\qquad$
b. Highest median: $\qquad$ What age did you use? $\qquad$
12. You can also change the mean or the median by adding a case to the data set. Pick an age that will raise the median. Add a case with that age: Choose the Add Case tool and click in the plot over the age that you want. A new case will be added.

What age did you add to raise the median? Why? How much did the median change?
13. What did you learn about means and medians from this activity?
14. Which measure-mean or median-do you think is more useful for summarizing the data about the ages of this group of people who got scooter injuries? Why?

## OVERVIEW

This exploration is designed to build students' understanding of the differences between the mean and the median. Students learn that the median is less affected by extreme values than the mean. For this reason, the median is a good choice for describing what's typical for data sets that have extreme values. The mean tends to be the first choice for students, perhaps because they are more familiar with it (through grades, sports, etc.). This lesson helps students to consider when the median might be a better choice.

## Objectives

- Learn about the concept of an outlier in an informal way
- Explore the effects of an outlier on the mean and median
- Deepen understanding of differences between the mean and the median, particularly that the median is less affected by extreme values than the mean
$\xrightarrow{\text { TinkerP价s }}$ Class Time: One class period


## Materials

- How Do Outliers Affect Means and Medians? worksheet (one per student)

Data Set: Scooters.tp (data for 28 people with scooter injuries)
TinkerPlots Prerequisites: Students should be familiar with intermediate graphing.

TinkerPlots Skills: Hiding cases and changing case values are explained in this lesson.

## LESSON PLAN

## Introduction

1. Introduce the task by connecting to the previous lessons. When your students looked at the ages of the people with scooter injuries, they probably made comments about the 41-year-old woman because she is so much older than the other people in the data set. Explain that this case is called an outlier because the value for age is unusually large in comparison to the other values.

Note: At the middle-school level students work with an informal definition of outlier: a value that is unusually large or small in comparison to the other values in the data set. Statisticians use formal methods for determining outliers. Using one definition, a value is considered an outlier if the distance from the value to the first quartile or third quartile (whichever is nearer) is more than 1.5 times the interquartile range.
2. Introduce the question that students need to investigate: How does an outlier affect the mean and the median? Explain that students will have the opportunity to change the age of the 41 -year-old to see what happens to these measures. Ask the class to come up with the lowest and highest reasonable ages for someone riding a scooter.

## Exploration

3. Have students work on the task independently or in pairs. They begin by finding the mean and median ages for the data set with all 28 values (question 2). Then, they see what happens to these measures when they remove the data for the 41-year-old (question 4).

## Troubleshooting tips for hiding the case of the 41-year old:

- Note that you will hide the case only in the plot that is selected.
- When you hide a case, it is not included in the calculation of any measures in that plot.
- To hide multiple cases at once, draw a selection rectangle around them.

4. Students tend to get very engaged with the task of changing the age to see the effect on the mean and the median. They can try two different methods to change the age of the 41 -year-old: typing a new age on the data card and dragging the value in the plot. Directions for each method are provided on the worksheet. You may also want to do a whole-class demonstration.

Troubleshooting tips for dragging values:

- To get the effect of changing the mean and median as you drag values, it's important that the scale of the plot be locked. Doubleclick an endpoint and click $\mathbf{O K}$ to lock that endpoint. You can change
the endpoint value, but you don't have to. You do have to lock each endpoint separately.
- Note that you have to choose the Drag Value tool again every time you let go of the mouse. If you don't, dragging will rescale the plot as usual. To keep the Drag Value tool selected, hold down the Alt (Win) Option (Mac) key.
- If you drag a case off the axis, it will move into the stack of excluded cases and the axis will show that there's more data off the end.

5. As you circulate, ask students, "How can you get the median to change by changing the age of the 41-year-old?" [To change the median, you need to change the age to a value that is less than the original median of 10.5 years.] The lowest the median will go is 10 years old. Students tend to be surprised that the median doesn't change when they increase the age.

To explore this further, ask students to see if they can increase the median by adding a new case to the data (the data set will now have 29 cases). By adding a new case that is higher than the original median of 10.5 , the median will increase. Show students how to use the Add Case tool and ask them to investigate: What new age could you add to the data set to increase the median? (Choose the Add Case tool and click in the plot over the age that you want. A new case will be added with all the other values blank.) Another way to increase the median is to change one of the ages that is below the median to an age above the median. Ask students to explore the question: What happens to the median if you change the values of other cases besides the 41-year-old?

## Wrap-Up

6. Have a class discussion to help students clarify the differences between means and medians, particularly that medians are less affected by extreme values than means.

- Why did the mean change more than the median when you removed the 41 -year-old from the data set?
- What was the highest mean you could get by changing this one value? The lowest?
- What was the lowest median you could get by changing this one value?
- Why doesn't the median change when you increase this one age? What would you need to do to the data set to increase the median?
-What did you learn about means and medians from this activity?
- Which measure-mean or median-do you think is more representative of the group of ages in this data set? Why?


## ANSWERS

## How Do Outliers Affect Means and Medians?

2.-4. Answers for question 3 will vary.

|  | Mean <br> $(\mathrm{yr})$ | Median <br> $(\mathrm{yr})$ |
| :--- | :---: | :---: |
| Data set with all 28 cases | 11.75 | 10.5 |
| Data set without the 41-year-old | 10.67 | 10 |
| How much did the measure change? | 1.08 | 0.5 |

5. Sample student work:

Jessica: Because outliers don't effect the median as much as they do the mean. So, when you take away the outlier the median doesn't have much to change while the mean does.

Eana: because the mean is the average, so with one less that is 41 yrs old it goes much lower. But for the median its not much because it's the middle \#,

SuJung: Because4iraisedthetotalalot andwhenitwasgoneit showed the true mean of the data.
7. Answers will vary.
9. Students should try ages in all parts of their chosen range (of youngest and oldest ages for riding scooters), but not beyond it. For example, changing the age to 30 yr gives a mean of 11.3 yr and a median of 10.5 yr . Changing the age to 5 yr gives a mean of 10.3 yr and a median of 10 yr .
10. a. Answers will vary depending on the student's choice for lowest age. A lowest age of 3 yr gives a mean of 10.39 yr .
b. 10 yr ; any age below the original median of 10.5 yr will give this value.
11. a. Answers will vary depending on the student's choice for highest age. A highest age of 60 yr gives a mean of 12.43 yr .
b. 10.5 yr ; any age above the original median of 10.5 yr will give this value.
12. Any age above the original median of 10.5 yr will raise the median. If the age is between 10.5 and 11 yr , the new median age will be the same as the age added. If the age is above 11 yr , the new median age will be 11 yr .
13. Sample student work:

Jessica: I learned that means can be greatly altered by an outlier while medians are not.

Justin: Medians don't change much, but means change drastically.
Eana: That each age changes the mean, the higher the age, the higher the mean, but for the median most of the time it stays the same.

Ian: The median depends on the \# of \#s in a data set while the mean depends on the size of the \#s.
14. Sample student work:

Jessica: Median: because even though the mean is the average, it can be thrown off by outliers, while the median (the middle number) is not.

Ian: Median because with the mean the data could be off because of a few odd balls.

Comments: These students recognize that the median is a good choice for representing a data set that includes outliers.

Rand: Mean because it sums up all of the ages, not just the middle one.
Comments: This student seems to have a misconception that the median is not representative of the group.

Nicole: Mean. I chose mean because it is more accurate. The median is just the middle number, and is usually the same as the mode.

Comments: This student seems to share the misconception with the previous student that the median doesn't represent the group. She has an additional misconception that the median is usually the same as the mode.

Justin: Mean. Median doesn'ttell much. The median is judged by what's above and below not that number.

Comments: This student seems to have a similar misconception about the median.

Yom: Mean because mean changes if one number is change but median doesn't. It is more specific than median.

Comments: The student has chosen the mean because she thinks it is more "specific." She is not focusing on how representative the mean is of the data set.

This activity is taken from Digging into Data with TinkerPlots, Key Curriculum Press, 2008.

In this activity participants look closely at two representations of the same ratio of two attributes: a scatter plot of the two attributes and a line plot (dot plot) of the ratios. How are the positions of points in these plots related? This activity requires the document Hall of Fame.tp from the TinkerPlots Online Resource Center.

1. Introduce the activity by setting the context: Participants will look at data about players in the Basketball Hall of Fame. They will look at three different ratios: points per game, percentage of successful free throws, and percentage of successful field goals. Make sure participants understand the definitions of free throw and field goal.
2. This lesson has two worksheets, Scoring Points and Free Throws and Field Goals, and will probably take participants about 30 minutes. If participants finish early, encourage them to postulate and explore other relationships in the Hall of Fame data set.
3. Use these questions (which are from the lesson plan) to summarize the activity:

- How can you figure out a basketball player's Points_per_Game ratio by using only the scatter plot?
- How does the information you get from the scatter plot of Free_Throw_ Attempts and Free_Throws_Made compare with the information in the line plot of Free_Throw_Percent?
- Are these players more successful at shooting free throws or field goals? How can you tell by comparing the scatter plots?
- In this lesson we saw a relationship between the positions of points on a line plot and their positions on a scatter plot. Would any line plot and any scatter plot have this relationship?


## Scoring Points

Name:

You will use two different plots to analyze the relationship between games played and points scored for basketball players in the Hall of Fame.

1. Open the TinkerPlots file Hall of Fame.tp to see the data on 75 players who are in the Basketball Hall of Fame.
2. The attribute Points_per_Game is the average number of points the player scored per game. It is the ratio of Points_Total to Games_Total.

$$
\text { Points_per_Game }=\frac{\text { Points_Total }}{\text { Games_Total }}
$$

For example, if you scored 200 total points and had played 10 games, you would have scored 20 points per game.
a. Make a line plot of Points_per_Game. Find the dot for W. Chamberlain, who has a Points_per_Game ratio of 30.1.
b. Find the dot for S. Martin. He played a total of 745 games and scored a total of 7337 points. What is his Points_per_Game ratio?
$\qquad$
3. Use the dividers and percentages to find the middle $50 \%$ of the data. What are typical values for Points_per_Game for this group of players?
$\qquad$
4. On a new plot, make a scatter plot with Games_Total on the horizontal axis and Points_Total on the vertical axis.
5. Use the Drawing tool to draw a line of fit on the scatter plot. Draw the line so that about half the points are above the line and about half are below it.

For the next two questions, you will select points in the line plot and see where they appear in the scatter plot. For example, if you select the four lowest dots on the line plot, TinkerPlots will automatically highlight those points on the scatter plot.

## Scoring Points

continued


6. Where do you think the middle $50 \%$ of the Points_per_Game values will appear on the scatter plot?
a. Guess first. What is your guess? $\qquad$
b. Select all the dots in the middle $50 \%$ on the line plot. Where are those points on the scatter plot? $\qquad$
7. Where do you think the top $25 \%$ of the Points_per_Game values will appear on the scatter plot?
a. What is your guess? $\qquad$
b. Highlight the top $25 \%$ on the line plot. Where are the points on the scatter plot? $\qquad$
8. Add each new point to both plots above and label the points.
a. Player H has a higher Points_per_Game ratio than most players.
b. Player L has a lower Points_per_Game ratio than most players.
c. Player T has a typical Points_per_Game ratio.
9. Which three basketball players are the best at scoring points? Where are these players on each plot?

## Free Throws and Field Goals

## Name:



You will be investigating data on free throws and field goals in basketball. A free throw is a penalty shot.

- Free_Throw_Attempts is the number of free throws tried.
- Free_Throws_Made is the number of successful free throws.
- Free_Throw_Percent is the percentage of free throws that are successful.

A field goal is any successful shot, except a free throw, that is made during game play.

## FREE THROWS

1. Open the TinkerPlots file Hall of Fame.tp to see the data on players who are in the Basketball Hall of Fame.
2. Make a line plot for the attribute Free_Throw_Percent.
a. If you made 30 free throws out of the 40 free throws you attempted, what is your free throw percent (Free_Throw_Percent)?
$\qquad$
b. How many players have a Free_Throw_Percent that is less than 65\%? $\qquad$
c. What are typical free throw percentages for this group of players? Use evidence from the data.
3. Make a scatter plot with the attribute Free_Throw_Attempts on the horizontal axis and Free_Throws_Made on the vertical axis.
$\theta$
4. Draw a line of fit on the scatter plot.

## Free Throws and Field Goals

continued
5. Experiment with highlighting points on the line plot to see where they appear on the scatter plot.
a. Where do the middle 50\% of the Free_Throw_Percent values appear on the scatter plot ? $\qquad$
b. Where do the bottom $25 \%$ of the Free_Throw_Percent values appear on the scatter plot? $\qquad$

## COMPARE FREE THROWS AND FIELD GOALS

Free throws


Field goals

6. Does this group of players tend to be more successful with free throws or field goals? How can you tell from the scatter plots?
7. For this group of players, a typical Free_Throw_Percent value was between $75 \%$ and $81 \%$. What are typical Field_Goal_Percent values for the group? Make a prediction and then use TinkerPlots to find out. Explain how you made your prediction.

# Session 4 Bringing TinkerPlots into the Classroom 

## LEARNING GOALS FOR THIS SESSION

Consider how best to use TinkerPlots activities and demonstrations in support of the subjects you teach.

## PEDAGOGICAL REFLECTION QUESTIONS

The object of this discussion is to consider the specifics of how to make the most effective use of TinkerPlots in your own classroom. Discussing these questions with other participants will help you clarify your thinking and develop strategies for making TinkerPlots a valuable part of your teaching. The questions below may be useful. However, the discussion should be guided by the concerns of the group.

- How does using TinkerPlots activities or presentations complement other hands-on activities or other approaches to the content (if it does)?
- What benefits will TinkerPlots provide to my students?
- What challenges must I overcome to get the maximum benefit from TinkerPlots for my students? (Consider facilities and equipment, classroom management, students with special needs, your own knowledge and confidence, and so forth.)
- What are the advantages of doing activities as whole-class presentations? Of doing them with student pairs?
- How can I use TinkerPlots to make opportunities for students to do mathematics, science, or social studies? How does doing help students make sense of concepts?
- How can I collect and assess the work students do with TinkerPlots?


## SUPPORTING MATERIALS

- Using TinkerPlots in Different Classroom Settings
- Methods of Collecting Student TinkerPlots Work
- TinkerPlots Reports and Presentations


## RESPONSE TO PEDAGOGICAL REFLECTION QUESTIONS

## TO THE PRESENTER

This discussion probably will take about 30 minutes. As the moderator of the discussion, make sure everyone's concerns are addressed. As needed, point out the resources included in this guide.

## Using TinkerPlots in Different Classroom Settings

You can use TinkerPlots in a number of different ways with your students, ranging from having each student explore data individually to using one computer hooked to a projection device to analyze data as a whole class. However, whenever possible, encourage students to discuss among themselves the "story" of the data: what the data are, how they were collected, how particular attributes were probably measured, what questions they have about the data, and what they think they will find. After they've analyzed the data, allow time for students to share and discuss what they have found, encouraging them to use the graphs they've made to support their conclusions. Trying to convince each other, using the data, will help students develop critical thinking skills and show them the value of supporting claims with credible evidence.

In a computer lab. Copy the activity you want students to complete and hand each student a copy to take to the school's computer lab and complete there. Alternatively, students can work in pairs on an assignment together. We've found it beneficial to have each student at a computer even when they are working cooperatively, so that each of them can easily take control of the program. It can be difficult to follow what another person does to make a graph, because things can happen quickly in TinkerPlots.

As a class. Hand out the activity to students and have them complete the Think About It section at home the night before or as part of a discussion with a partner or small group. Then as a class discuss the various answers students came up with. Sharing their expectations about what they will see makes it clear to students that different people expect different things. This diversity makes them all the more eager to see what the data actually say. Then work as a class to make a graph (or several graphs) that answer the question on a single computer hooked to a projection device. Again have different students make suggestions about what to do, and what the graphs mean.

Part of the reason some students don't need much instruction in using TinkerPlots is that they are quite familiar with technology. Some even have more experience, and much less fear, than their teachers. And if allowed to collaborate and share with one another what they have just learned about the software with one another, students are remarkable. In a large computer lab, you won't need to tell students to collaborate. Often they will do it naturally. One student will figure out how to do something new, and very quickly, that know-how will spread around the room. Some students will quickly become the local source of TinkerPlots know-how, and we've observed that these are often the very students who otherwise seem lost or uninterested in class. If there is something you want to introduce into your students' repertoire, find a suitable time to show one or two students. Again, the know-how will spread.

## Methods of Collecting Student TinkerPlots Work

| Method | Advantages | Disadvantages |
| :---: | :---: | :---: |
| Viewing student work on their monitors, perhaps checking it off as you go around the room | - Very simple, and you'll be doing it anyway | - Minimal written record <br> - Often you don't get to every student during the class period |
| Collecting completed photocopies of activity worksheets | - Easy to collect and assess <br> - Worksheets are wellstructured to provide guidance to students | - No plots <br> - Worksheets can lack flexibility and don't allow as much exploration |
| Collecting printouts of student files | - Easy to collect and assess | - Depends on your printer availability; having the whole class trying to print at the end of the period is not practical |
| Collecting work in student file folders on a network | - Easy to collect | - Can be timeconsuming to assess multiple files <br> - Requires a network |
| Collecting student work on one or or more portable storage devices | - Doesn't require access to a network | - Collection process can be complicated <br> - Can be timeconsuming to assess multiple files |

## TinkerPlots Reports and Presentations

Sometimes you might want students to write a complete report of an investigation or another activity. The document Backpack Report.tp in the Data and Demos Demos folder is a sample student report for the first activity you did. Note that this document contains live graphs as well as static pictures and text boxes.

## TEXT BOXES

To create a text box, drag a Text icon from the toolbar into the document and start typing. You can format text using the Text menu.

PICTURES
You can copy TinkerPlots objects as pictures and paste them into both TinkerPlots documents and other applications such as Microsoft Word.

1. Select the object and choose Edit | Copy As Picture.
2. Click in a blank space in the document (in either application) and choose Edit $\mid$ Paste.

To paste a picture from another application into a TinkerPlots document:

1. Copy the picture to the clipboard.
2. In TinkerPlots, click in a blank space in the document and choose Edit $\mid$ Paste.

## LEARNING GOALS FOR THIS SESSION

Explore resources provided with TinkerPlots that you can use to help your students achieve your learning goals.

## TINKERPLOTS MOVIES

TinkerPlots comes with five movies. To watch one, choose TinkerPlots Movies from the Help menu. Each movie is about 5 minutes long. TinkerPlots Basics is a good introduction for both teachers and students. After watching this movie, most students can go right into working with data. Adding Data shows three different methods to get your own data into TinkerPlots. Making Common Graphs, Comparing Groups, and Exploring Relationships all show different ways you can use TinkerPlots to help students understand data.

## DATA AND DEMOS

TinkerPlots comes with about 40 data sets ready for you and your students to use. Go to the Data and Demos folder (in the TinkerPlots application or program folder) and open a subfolder (Health, Science and Nature, Social Studies, or Sports and Entertainment). Each data set contains one or more questions for students to explore. We suggest you have students treat these like any investigation: Make a hypothesis, analyze the data, and then communicate their conclusions.

The Demos folder contains nine demonstrations showing more advanced capabilities of TinkerPlots. These are for teacher use, although some are meant to be modified and presented to students.

## TINKERPLOTS HELP

Find out how to do all sorts of things with TinkerPlots by choosing TinkerPlots Help from the Help menu. This workshop session includes a TinkerPlots Help Scavenger Hunt that you can use to familiarize yourself with Help and to learn about some things you might not have known you could do. You can also use the TinkerPlots Checklist at the end of this guide as a quick reference for what you can do.

## RESEARCH ON STUDENT THINKING ABOUT DATA

This article, from the book Exploring Data with TinkerPlots, describes recent research that influenced both the design of TinkerPlots and the types of data and activities included with it. The article appears on page 60 of this guide.

## EXPLORING DATA WITH TINKERPLOTS

This book comes with all full editions of TinkerPlots and contains seven activities for use in grades 4-8 as well as notes on getting started, learning TinkerPlots, and teaching with TinkerPlots. See the activity notes for the backpacks activities in this book for some discussion of what to expect from younger and older students. A PDF file of this book is available in the TinkerPlots Help folder (in the TinkerPlots application or program folder).

## TINKERPLOTS ONLINE RESOURCE CENTER

At www.keypress.com/tinkerplots/resources, you can find links to websites with data as well as share activities and data sets with other teachers.

## TO THE PRESENTER

This final session is designed to allow you and workshop participants some flexibility in what to focus on. You can have participants explore these options independently, or you can have the whole group complete one or more of the activities. For example, you might watch one or more of the movies as a group, or have participants work with a particular data set in the Data and Demos folder.

## TinkerPlots Help Scavenger Hunt

Several advanced capabilities of TinkerPlots are listed on these pages. Your task is to learn how to do as many of them as you can using TinkerPlots's online Help system. (If you already know how to do one of these, don't bother looking it up in Help.)

To open Help, choose TinkerPlots Help from the Help menu. You can search for items using the Contents, Index, or Search functions at the left of the Help window. The Help system sometimes refers to the upper toolbar and the lower toolbar. The lower toolbar is located at the bottom of the plot window and includes the Mix-up button and icon types. The upper toolbar is the collection of buttons at the top of the document window and includes the Separate, Order, and Stack buttons.

1. Change the color of values of a category attribute. Here the color for males was changed from yellow (left) to the stereotypical blue (right).

2. Remove an attribute from an axis without using the Mix-up button or undoing. In this example, the attribute Grade was removed from the vertical axis (left) to graph PackWeight all by itself on the horizontal axis (right).


3. Make a pie graph. This pie graph shows there are about equal numbers of that males and females in the backpack data set.

4. Set the axis endpoints and interval widths to specific values. In this example, the axis showing BodyWeight that TinkerPlots automatically gave (left) was changed to start at 0 and have a bin width of 10 (right).

5. Make a filter to hide some cases in a plot. This plot was filtered to show the data for only the third graders.


During the last ten years, statistics educators have learned quite a bit about how younger students learn to reason about data and how experts approach data differently from ordinary people or novices. This research influenced both the design of TinkerPlots and the type of data and activities included with it. Here we briefly describe some of what we have learned from research and how we've applied it to TinkerPlots. For a more detailed description of this research, see Chapter 13 of A Research Companion to Principles and Standards for School Mathematics (2003) published by the National Council of Teachers of Mathematics (NCTM). Other research articles are listed in the TinkerPlots Online Resource Center at www. keypress.com/tinkerplots.

## DEVELOPING AN AGGREGATE VIEW OF DATA

Experts look at a group of data as a collection, or aggregate. When they look at a histogram or stacked dot plot, for example, they tend to see its general shape, how spread out it is, and where the data are centered. When experts compare two groups (say two different stacked dot plots showing people's heights) they generally base those comparisons on these aggregate features. For example, an expert looking at two distributions of height for men and women might conclude that men tend to be taller than women by comparing the locations of the centers (for example, the means) of each distribution.

Students new to the study of data tend not to see these aggregate features. Rather they see individual cases or subgroups of individuals with the same, or nearly the same, value. Looking at the same distribution of height, students might focus mostly on individuals on the extreme ends of the distribution. Alternatively, they might tend to see short people, average people, and tall people, and reason about these as three different groupings. When students compare two groups, they often compare the number of cases at one of the extreme ends of the distributions. For example, a student looking at a graph showing men's and women's heights might argue that the men are taller because there are more of them above 6 feet. One problem with this comparison is that it doesn't take into account the number of cases in each group. If we had a group of 1000 women and 10 men, we would probably have more women than men above 6 feet. This method of comparing numbers in common slices of the distributions often leads to faulty conclusions.

Researchers have also found that it usually isn't fruitful to simply tell students that what they are doing is wrong and to recommend an alternative. If students don't understand why their methods are flawed and why the alternative is better, they will do what you ask now, but later revert to what makes more sense to them. This research led us to take a different approach in TinkerPlots. We have included tools
students can use to look at and compare graphs using their intuitive methods. For example, students can use dividers and reference lines to break up distributions into parts and click a button to quickly get counts of cases in different parts. We believe that by allowing students to use methods that make sense to them, and having them discuss what they do with other students and with teachers, they will come to see the limitations in their approaches and develop more powerful methods.

## DESCRIBING CENTERS OF DISTRIBUTIONS

Rather than using means and medians to summarize numeric data, researchers have reported that most people, including young students, prefer to describe a distribution using a "center clump": for example, "Most of the data are between 15 and 25." Dividers and hat plots are TinkerPlots tools designed specifically to help students describe data in terms of these informal averages. Students can also use the Drawing tool to show where they think the bulk of the data are. It is fine for students to summarize groups and to describe differences by saying that one group's center clump is higher up than another group's. We expect that formal averages, like means and medians, will then eventually make more sense when students see that these are almost always located within what they see as the center clump.

## STAYING GROUNDED IN THE DATA

Many researchers have reported the importance of students keeping in mind what a case is-that a data point on a graph is not simply a number or dot, but that it is a measurement for a case (for example, a person's height). Often when students (or experts) get confused looking at a graph, it is because they lose this connection between a case and a point on the graph. In TinkerPlots, students can, at any time, click a point on a plot to see the data card for that individual case and to automatically highlight that case in any other plots that may be open. Again, this helps students keep in mind what a case is. It also helps them analyze their data, because when students highlight a case that has a high value for one attribute, they can see where that case is located in another graph of a different attribute. For these reasons, in all TinkerPlots graphs-even histograms, box plots, and pie graphsit is easy for students to see cases and how those individual cases work together to form a particular graph. When one plot changes to another, TinkerPlots animates the transformation to help preserve the integrity of the case.

This list describes some of the many things you can do with TinkerPlots. Some you already know how to do, and some you may never need to do at all. If you like, use this list to keep track of your expertise with TinkerPlots. You can also scan this list to find out about some of the things you didn't know TinkerPlots could do and pick interesting ones to explore either through experimentation or through TinkerPlots Help.

## GETTING DATA INTO TINKERPLOTS AND READY FOR STUDENTS

- Enter data directly into a data card, a case table, and a graph.
- Rename a data set.
- Import data from the web, from Excel, and from Word.
- Add an attribute defined by a formula.
- Cut, copy, and paste attributes and cases.
- Duplicate an object.
- Delete an object.
- Show values in scientific notation.
- Round values using a formula.
- Sort cases in a table.
- Create realistic data using a random number generator.
- Combine data sets.
- Add multiple new cases at once.
- Lock a data set so that students can't change it.


## WORKING WITH PLOTS

- Add a drawing to your plot and remove it.
- Remove an attribute from an axis without using the Mix-up button.
- Make a pie graph, a value bar graph, and a rectangle graph.
- Set axis endpoints and interval widths to specific values.
- Lock the color in a plot.
- Turn off plot color and highlight cases in two or more plots.
- Drag values while keeping the axes stationary.
- Make a box plot.
- Divide a plot so that each part has the same number of cases.
- Remove some cases from a plot by filtering.
- Hide some cases in a plot.
- Change bin widths in a histogram by dragging lines.
- Reorder category bins.
- Label icons by your attribute of choice.
- Change the colors of categories and of a color gradient.
- Use a filter with a slider to animate a time series graph or to grow a sample.
- Make a value table by hiding icons in the plot.
- Connect the dots in a plot.
- Reverse axis endpoints.
- Use continuous-style bins.


## MAKING REPORTS OR PRESENTATIONS

- Make a picture of a graph.
- Import a picture from another application.
- Make a text box and format text.
- Make text in graphs larger.
- Show page breaks and print a document.
- Export data to another application.
- Iconify an object to save space.
- View an object in its own window.


## CUSTOMIZING DOCUMENTS

- Remove some plot features from the toolbars.
- Hide attributes in the data cards.
- Change image icons.
- Hide objects in the document.
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| Click to select the case in your |
| :--- |
| plots. Click outside to deselect. |$\quad$| Click to browse |
| :--- |
| cases. |






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