OVERVIEW

In this activity, students are introduced to the game Wink, Blink, and Stare and asked to determine whether it is fair. To do so, students first play the game themselves, and then use a sampler in TinkerPlots to model the game and collect more data. Although most students originally think the game is fair, they quickly observe that Wink occurs more often.

Students then work toward understanding why Wink is more likely—because there are two ways to get Wink and only one way to get Blink or Stare. They are introduced to the terms *simple outcome, combined outcome,* and *sample space.* Students then learn the relative frequency (or experimental) interpretation of probability—that Wink having probability $\frac{1}{2}$ means that if they conduct a large number of trials, they expect to get Wink about $\frac{1}{2}$ the time. Finally, students use TinkerPlots to play the game repeatedly, observing that a sample of 2500 gives them the results they expect more often than a sample of 100.

Activity Time: Three class periods

Objectives

- Identify aspects of what makes a method of selection "fair."
- Understand that when thinking about how likely certain events are, you have to consider the number of ways it can happen.
- Understand concepts such as sample space, expectation, and probability.
- Sketch expected results and compare them with actual results.

Common Core Standards Addressed

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

Grade 7, Statistics and Probability Standard 6

Represent sample spaces for compound events using methods such as organized lists, tales, and tree diagrams. For an event described in everyday language, identify the outcomes in the sample space which compose the event.

Grade 7, Statistics and Probability Standard 8b

Design and use a simulation to generate frequencies for compound events.

Grade 7, Statistics and Probability Standard 8c

Prerequisites

- Experience sketching the general shape of a distribution (perhaps by doing the activity Sketching Distributions)

Materials

- Wink, Blink, and Stare worksheet (one copy per student)

- Stickers (or something for prizes)
- TinkerPlots sampler filled with students' names (optional)
- Bag with five red balls and one green ball
- Bag with two chips, one marked with "•" and one marked with "–" (one bag and pair of chips per group)

LESSON PLAN

DAY 1

Choosing Fairly (10 minutes)

Choose five students to help you play the game. One way to do this is to use a sampler in TinkerPlots. Open a new document in TinkerPlots, drag a sampler from the shelf into the document, and enter the names of your students into the mixer. (It is helpful to prepare this document ahead of time. You can remove the names of absent students at the beginning of class.)

Change Repeat to 1 and change Draw to 5, then do a practice run, so students can see what happens. Click the **RUN** button to run the sampler, drawing five names from the mixer at slow speed. Students will probably discover that with the current setting (with replacement), the same name can come out twice. (You may need to draw five more names if this doesn't happen the first time.)

Ask students what is wrong. They should recognize that the names shouldn't be replaced after they are drawn. Show students how to change this setting by clicking the arrow at the lower left corner of the mixer and selecting **Replacement | Without Replacement** from the **Device Options** menu. Run the sampler to choose another five names, again to make sure it is working correctly. Then run the sampler one more time, this time to choose five volunteers.

Choosing One of Two Students Fairly (15 minutes)

Now that you have five names, ask the first two students that were chosen to come to the front of the class. Announce to the class that one of these two students will receive a prize, and ask all the students:

> *How can we choose fairly who will get the prize?*

Briefly discuss each method the students suggest, each time asking the class why it is fair (or not). You might want to make a list of their suggestions as well as a list of reasons a game or way of choosing is fair and a list of reasons a game or way of choosing is not fair.

Typically, students will suggest that a method is fair because the outcome can't be controlled or predicted; they will less often say that it is fair if the chances or probability of each student winning are equal. That's okay, as that aspect will be introduced later.

Hold up the bag containing five red balls and one green ball, but don't tell students how many of each color ball are in the bag. Announce, "To choose a winner, we're going to use this bag. The bag contains balls of two colors—red and green. We'll assign [first student name] red, and [second student name] green. Each time we draw out a red, [first name] gets a point. Each time we get green, [second name] gets a point. We will repeat 11 times and replace the ball we get after each draw. Whoever has the most points after 11 draws is the winner."

> *Is this game fair?*

Students might ask how many of each color ball are in the bag, but don't tell them. Instead, ask if it is important. Students might now introduce the idea of "equal chances" or "probability." If they do, add this to the list of what makes a game or way of choosing fair.

Play the game, keeping track of each student's points on the board, and award a prize. Then ask students whether the game was fair. Most students will suspect that the numbers of red and green balls in the bag are not equal. Show them that the bag has five reds and one green. If students ask before playing the game about the number of balls of each color, ask whether that matters and why, but don't show the contents of the bag until the end. If it hasn't been mentioned before, add criteria about "equal chance to win" to the list of reasons a game is fair.

Wink, Blink, and Stare Game (10 minutes)

Call up the remaining three students chosen. They will play a game called Wink, Blink, and Stare.

Announce that you have a bag with two chips in it. One disk has a big dot on both sides. The other disk has a dash on both sides. Show students the disks.

To play the game, you'll draw one chip out of the bag, two times. Draw the first chip out of the bag and record the result. Put the chip back in the bag, shake it, draw a second chip, and again record the result.

Explain to students that the result is considered a "Wink" if one chip drawn is a dot and the other is a dash, a "Blink" if both chips drawn are dashes, and a "Stare" if both chips drawn are dots. As you explain these terms, you might hold the chips up to your eyes to demonstrate the meaning of these events (• • looks like two eyes wide open). Assign each of the three students an event. If a student's event is drawn, that student wins the game.

On the board, write the three possible events and the student who wins next to that event.

Event	Winner
Wink: • –	
Blink: – –	
Stare: • •	

Conduct the drawing once and award a prize.

Distribute the Wink, Blink, and Stare worksheet. (You might want to hand out only the first page now, and save the remainder for later.) After students have completed Steps 1 and 2, invite them to share their thoughts and discuss.

If someone were to think this wasn't fair, what could you do to demonstrate that it was?

How about if we were to play this game many times and keep track of how many times each player won? Would this be a way to decide whether this game was fair or not?

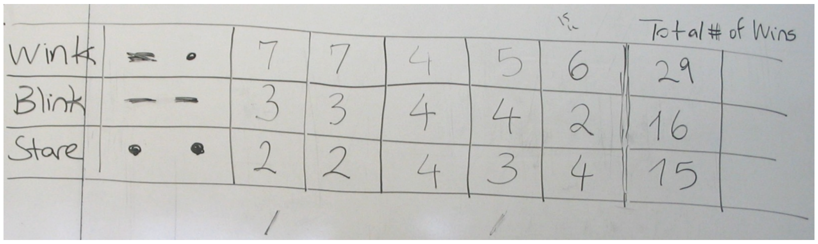
Suppose we were to play it 12 times and keep track of what happened. How often would you expect each player to win if the game is really fair?

Small Groups: Playing the Game 12 Times (15 minutes)

Have students work in pairs to complete the first page of the worksheet, playing the game 12 times and recording the results. After they have conducted and recorded 12 trials, record the results of each group on the board or overhead as shown. Then ask:

Is the game fair? Why, or why not?

Would playing the game many times, and keeping track of the results, help us decide whether the game was fair?



							Total # of Wins
Wink	– •	7	7	4	5	6	29
Blink	– –	3	3	4	4	2	16
Stare	• •	2	2	4	3	4	15

This table shows the results of five groups, each of which played the game twelve times. The right-most column, suggested by students, shows the combined totals for each event.

Ask students what they conclude from these data. You might want to have a discussion about the variability of the data.

At this point, many students will suspect that Wink, Blink, and Stare is an unfair game, but may not be able to explain why.

Before sending students to computers, ask students about their predictions if they were to play the game 100 times. Some might still think that each result would occur the same number of times, although some might begin to formulate how much more likely a player is to get a Wink than to get a Stare or a Blink.

If we play the game 100 times, how often do you think each result will occur?

Day 2

Computer Modeling (10 minutes)

Hand out the rest of the student worksheet. If this is the students' first experience using the sampler, you may first want to work together as a class to build and run a model. Open a new document and drag a sampler into the document. Edit mixer elements to be a dash (–) and an asterisk (*), and if you have an extra element, click the **–** button to delete it. Double-click *Attr1* and change it to *Draw1*. Change *Attr2* to *Draw2*.

Check that Draw is set to 2 and click the **RUN** button to run the sampler. Ask students if this sampler can be used to play the game instead of the bag containing chips. If a student suggests that it cannot, ask them what is different. If students are familiar with TinkerPlots, you might instead suggest that each pair play the game 100 times using their bag and chips. Students are likely to protest and suggest using TinkerPlots instead. Have students describe how to do this. When they have described how to set up the sampler, ask students if this will give the same results as using the bag. This requires them to defend the computer model in order to avoid playing the game 100 times by hand.

Student Work at Computers (20 minutes)

Have students move to computers and complete Steps 6–13. In Step 12, students create a plot. If they don't have experience making plots in TinkerPlots, they may need some guidance. There are several ways to do this. One is to drag *Draw1* from the results table onto one axis, and *Draw2* onto the other axis. Then click the **Count (N)** button in the upper plot toolbar to show the count for each category. Alternatively, drag a plot into the document, drag the *Join* attribute from the results table onto the horizontal or vertical axis, drag a point to separate the values into bins, and then click the **Count (N)** button.

Explaining Why the Game is Not Fair (20 minutes)

Ask students what they have learned from running their model. By now, most students will be convinced that Wink occurs more often than either Blink or Stare. Ask students if they can explain why.

If students say, “because it just happens more often,” ask:

> *Suppose Marco still believes that the game is fair. He argues that because you get different results every time, you can’t decide whether it’s fair or not by playing the game. How would you convince him the game is not fair?*

Once students play the game, they often really want to understand why they get the results they do, so give students ample time to think about why and to offer and discuss their arguments. A few students are likely to have a sense of why, but may not be able to articulate it. However, with some thought, one or more of the students may be able to explain it to the satisfaction of others—convincing other students that there are two ways to get Wink and only one way to get Blink or Stare. There are hints available in the TinkerPlots graph which they may have picked up on. They may have noticed that they can split Wink into two bins, • — and — •, or that the bin that combines the two outcomes has icons with two different colors, while the bins for Blink and Stare have icons with only one color.

If students do not come up with a convincing argument, play the game a few more times, using the bag and chips. You be Wink. Pick two students, one to be Blink and the other Stare.

Ideally, someone will notice that as soon as the first chip is drawn, either Blink or Stare is out of the running, but Wink is always still in the game. If students don’t notice this, it may help to have whoever is out of the game after the first draw sit down. They may now be able to see why Wink will win about half the time. Wink always survives the first draw, and then goes on to win half of the time on the second draw.

Once students notice this pattern, ask them why they think this happens.

> *Is this why Wink has a better chance?*
>
> *What are all the different things that can happen when we play this game?*

Write the possible outcomes on the board as students name them. With the four individual outcomes written on the board, explain that these are called *simple outcomes.* The complete list of all possible simple outcomes that can occur is called the *sample space*. Then, combine *, — and —. *, giving the two outcomes the name Wink. Explain that because Wink is made up of two *simple outcomes,* it is called a *combined outcome.* You might want to write these terms on the board, as they are concepts that students will need frequently.

Day 3

Introduce Probability (20 minutes)

Once students understand that there are two ways to get Wink, and only one way to get Blink or Stare, use these questions to introduce students to probability as a measure, or value:

If you play the game 100 times, how many Winks do you expect to get? How many Blinks? How many Stares?

If you play the game 200 times, how many Winks do you expect to get? How many Blinks? How many Stares?

As a class, run the sampler with Repeat set to 200.

Is this about what you expected?

Run the sampler a few more times, stopping after each run to evaluate whether the results are close to what students expect. Then ask:

How did you come up with the number of Winks you expected if you played the game 200 times?

Students may say something like "Well, you expect Wink will win half the time, so half of 200 is 100."

Tell students that instead of saying that Wink wins about half the time, they can instead say that the probability of Wink winning is $\frac{1}{2}$, or 50%.

What's the probability of Blink winning? (1 out of 4, $\frac{1}{4}$, or 25%)

Note to students that when we say Blink has a 25% chance of winning, what we mean is that if we play the game over and over, Blink will win about 25% of the time.

You may wish to expand this to another common situation, like rolling a die or drawing a card from a standard deck of cards. Be sure that students understand that the term *probability* is used to express what is expected to happen over many trials, but this does not mean that the results will always match our expectations. They almost never do.

Compare Results Versus Expectations for Two Sample Sizes (20 minutes)

Have students consider the results they expect if they play the game 100 times and sketch a graph of the results. Remind students that when sketching, they want to sketch the basic shape, without the fine details. After they sketch what they expect with 100 games, run the sampler with Repeat set to 100 for the class. You might demonstrate how to use the icon type **Fuse Rectangular** and how to arrange the three events in order along the horizontal axis. Demonstrate making a quick sketch of your results on the board and rate how well the results match the expected results. With a sample size of 100, results that look quite different than the expected graph occur quite often.

After students have completed the worksheet, bring them together to report their findings. The results using a sampler of 2500 will be closer to the expected results. This is an important feature of sampling. We expect Wink 50% of the time, but if we conduct 10 trials, it's easy to observe Wink occurring anywhere from 30% to 70% of the time. When we look at 100 trials, however, Wink will tend to occur from 45% to 55% of the time. By the time we look at the results of 2500 trials, Wink will tend to occur from 49% to 51% percent of the time. This helps students see why larger samplers are better.

ANSWERS

1. • •, • –, – –. Students will probably not list • – and – • as two different possibilities.
2. Students will likely think that it is fair.
4. Students' results will vary quite a bit, but Wink will consistently win more often, convincing most of them that the game is not fair. Many students will conclude that Wink is twice as likely as Blink and Stare, but they probably won't be able to explain why.
5. Because Wink is expected half the time, and Blink and Stare are each expected a quarter of the time, the graph should have bars of equal heights for Blink and Stare, and a bar for Wink that is twice as high. Some students might purposely make the bars for Blink and Stare unequal in order to communicate that they would be surprised if they came out exactly equal. It is true that the exact results don't occur very often, but neither do any exact results we draw. Be sure students understand that when they express the results they "expect," it is understood that the results will not necessarily come out exactly as expected.

16–19. Students results and comparisons will vary. For most students, the sample of 2500 will match their expectations more closely than their sample of 100.

Most students will notice that sample of 2500 is closer to their expectations than their sample of 100. As a class, you might look at a sample of 20 games using TinkerPlots. Ask students how close they think the results will be to their expectation.