OVERVIEW

In this activity, students explore various outcomes associated with rolling two dice. They start by playing the Two-Dice Elimination game, in which they choose a sum they think will be least commonly rolled. Then students simulate rolling two dice many times using TinkerPlots to determine whether a *step model* or *triangle model* of the distribution of sums is correct. Finally, they use the triangle model to calculate the probability of rolling each sum.

Students build a sampler, add attributes to a results table, and plot results. It is helpful if students already have some or all of these skills. If students don't have these skills yet, you can demonstrate for them, or you can do Steps 7–12 as a class demonstration. Specifics of how to do each of these actions are not provided in these Notes, but you can learn how to do them by watching the TinkerPlots movies "TinkerPlots Basics" and "Probability Simulation."

Activity Time: Two class periods

Objectives

- Choose, based on data, the correct sample space for rolling two dice.
- Create a model to simulate a random process—rolling two dice and summing them.
- From a sample space, determine the probabilities of compound events.
- Understand why, in certain cases, it is important to consider the order in which simple outcomes occur.

Common Core Standards Addressed

Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

Grade 7, Statistics and Probability Standard 1

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates of predictions.

Grade 7, Statistics and Probability Standard 2

Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

Grade 7, Statistics and Probability Standard 7b

Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

Grade 7, Statistics and Probability Standard 8

Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

Grade 7, Statistics and Probability Standard 8a

Prerequisites

- Familiarity with sketching distributions and recognizing common distribution shapes (perhaps by doing the activity Sketching Distributions)
- Have some understanding of the importance of order when exploring a sample space (perhaps by doing the activity Wink, Blink, and Stare)

Materials

- Two dice
- Sum of Two Dice worksheet (one per student, ideally with page 1 copied separately from pages 2–4)
- **Sum of Two Dice.tp**

LESSON PLAN

Before Class Begins

Make a TinkerPlots document that contains the first names of all your students, with a sum of two dice between 2 and 12 assigned to each name, and a plot of students' assigned numbers. (They'll choose their own numbers in class, so it doesn't matter what you assign them here.) **Sum of Two Dice.tp** shows how this may look once it is set up; you could open this document and replace the student names there with your students' names. Or, create a collection in TinkerPlots, drag a case table into the document, name the first attribute *Name,* and enter student names in this column. Then create a second attribute called *Sum.* Assign values from 2 to 12 to each student. Drag a plot into the document, and plot *Sum* on the horizontal axis. Double-click the axis end values and change them so the axis shows values from 2 to 12. Set the Bin Width to 1. With *Name* selected, click the **Label** button in the upper plot toolbar to display students' names next to their icons. (The option **Labels Above** will work best.)

Day 1

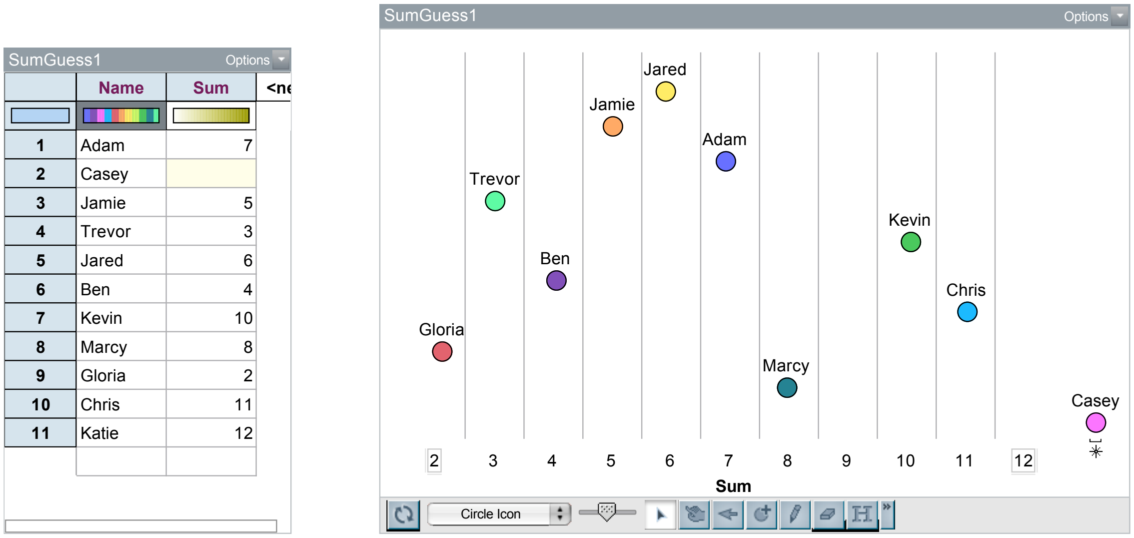
Two-Dice Elimination Game (20 minutes)

Display the TinkerPlots document you created before class.

Introduce students to the Two-Dice Elimination game. To play the game, have each student choose a sum of two dice.

Select the **Drag Value** tool  in the lower plot toolbar. As each student chooses his or her number, move the student's point to the selected sum using this tool.

Roll two real dice. Any student who selected the sum of the two dice is eliminated from the game. You can show this in TinkerPlots by deleting his or her value for *Sum* from the case table. This will cause the student's icon to move to the Missing Value column to the right of the horizontal axis. Continue until only one number remains.



Play the game again. Have each student again select a new sum, one he or she thinks will "survive." Reposition each player again to the sum he or she chooses in the plot. You might want to ask a few students to explain their choices. After a brief discussion, allow students to change their sums, but only if they explain why they want to switch.

Now that you have played the game twice, help students express the idea that there are multiple outcomes that produce the middle sums, but few that produce either high or low sums. (Don't worry yet about whether 1, 6 is the same or different from 6, 1.)

Which numbers are best and why?

Why might some sums be more or less likely than other sums?

The goal is not to get students to change beliefs based on the sample (because it's too small); rather, it is to get them to express their ideas about the probability of each sum based on the number of ways each sum can occur. Students who have done the activities "Wink, Blink, and Stare" and "Four-Child Families" may make this connection quickly and suggest that the number of ways you can get various sums is important.

Thinking About the Sample Space (20 minutes)

Hand out only page 1 of the worksheet. Have students complete Step 1, making sure they understand the question.

After each student has chosen a distribution, discuss as a class, and encourage students to explain why they expect their chosen shapes. If they understood the game they just played, students should expect lower frequencies for values toward the extremes, but they may not yet expect a difference in the frequency of occurrence of sums toward the middle (sums of 6, 7, and 8, for example). Don't expect or suggest (or even validate) a correct answer at this point; just have several students explain their thinking.

Next have students complete Step 2. If students have studied tree diagrams, you may want to remind them that this could be a useful tool.

Once students have completed Step 2, hand out the rest of the worksheet. Students should work individually to complete Steps 3–6. Once they have completed this task, they will have a chance to explore simulated data using TinkerPlots.

Day 2

Student Work at Computers (20 minutes)

Explain to students that they will build a model that simulates for rolling two dice in TinkerPlots. They'll then run the model and observe the shape they tend to get for the distribution of sums in order to decide between the triangle model and the step model. If possible, display the two models on the board to remind students of their predictions. You also might want to remind students that when sketching a distribution, they should try to capture the basic shape of the graph rather than duplicate the details.

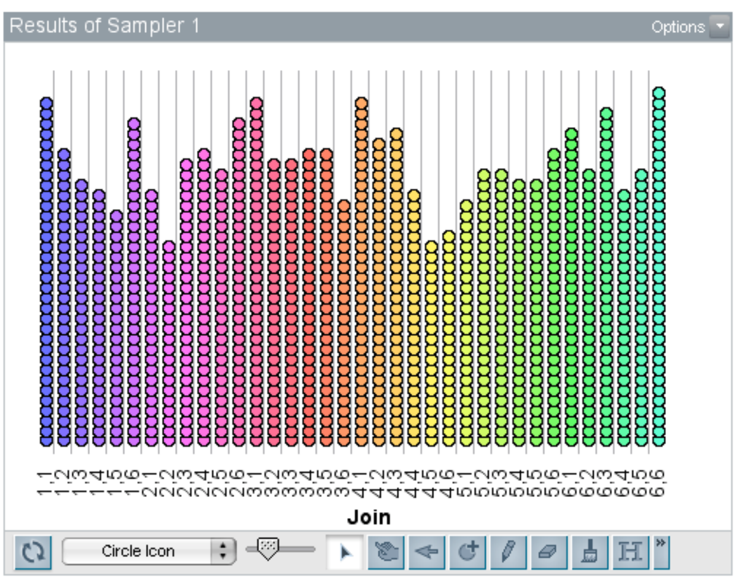
Have students complete Steps 7–12. If students are not familiar with TinkerPlots, you may want to do this as a class demonstration.

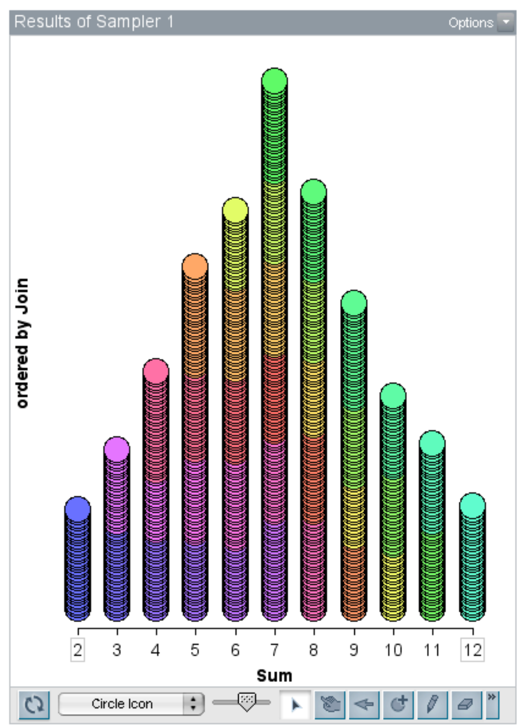
Discussion (15 minutes)

Once students have run their model in TinkerPlots, pose the following questions for class discussion.

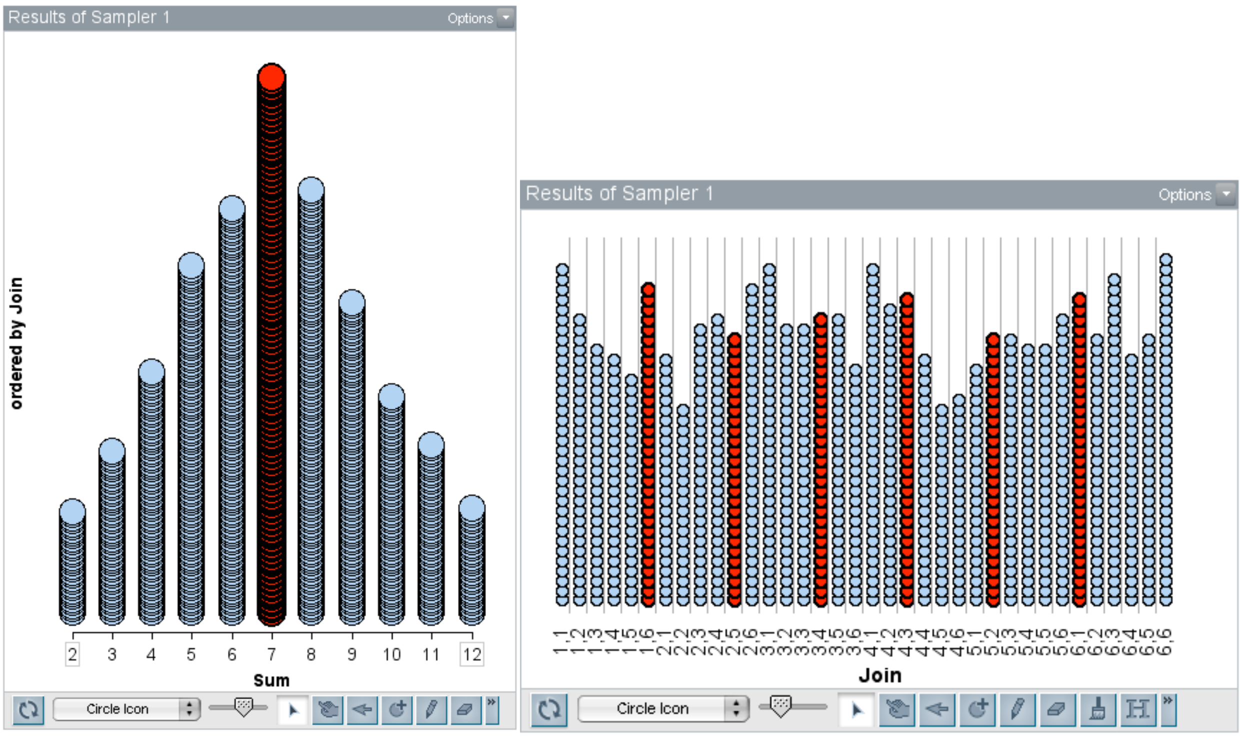
> *Which model appears to be correct, the step model or the triangle model?*
>
> *Can you explain why the triangle model is correct? Why do we get more sevens than any other sum? Why do we get fewer twos and twelves than any other sums? Why do we get more sums of three than sums of two?*

If students are still struggling to explain why a sum of 3 occurs more often than a sum of 2, demonstrate the results of 1000 trials once more using TinkerPlots. Set up the document to show a plot of the simple outcomes (the *Join* values) and, in a second plot, the sums. Click the *Join* outcome in the results table to color the data points according to the outcome, and point out the number of different colors that make up each sum. See the examples here.





Then turn off the color gradient (click the light blue rectangle at the top of the first column in the results table), and highlight the cases with a sum of 11 in the *Sum* plot (click and drag to draw a selection rectangle around all values of 11). Observe that two different simple outcomes are highlighted in the *Join* plot. Next highlight each case with a sum of 7 in the *Sum* plot. Now there are six different simple outcomes highlighted in the *Join* plot, as shown here.

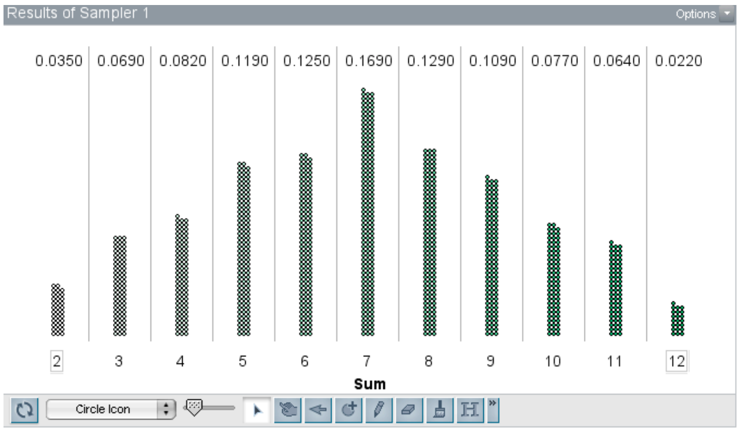


Another way to demonstrate the importance of order is to have students imagine that they rolled one die and got a 6. What would the second role need to be to get a sum of 11? A sum of 12? Now have them imagine they rolled one die and got a 5. What would the second role need to be to get a sum of 11? A sum of 12? A roll of 5 or 6 on the first roll leaves open the possibility of getting a sum of 11 on the second roll, but to get a sum of 12, you have to get a 6 on the first roll. So, there are two ways to get a sum of 11 (5, 6 and 6, 5) and only one way to get a sum of 12 (6, 6).

Computing Probabilities from the Sample Space (15 min)

Explain that the 36 simple outcomes in the triangle model form the sample space for rolling two dice. All possible simple outcomes are included in the list. Using the sample space, have students complete Step 16. Some students will need considerable support. You might assign different events to different students, so not everyone does them all. You may also need to review how to convert fractions to decimals.

After students have computed the probabilities in Step 16, reproduce the table shown in the answer to Step 16 on the board. Draw a sample of 1000 rolls and compare the proportions you get to the theoretical ones students computed using the sample space. In general, the proportions should be quite close to the theoretical values. You might want to call students' attention to one event (for example, a sum of 3) and monitor that value as you run repeated samples to get a sense for how the number varies. You could then reduce the sample size to 100 and watch the same value. It will tend to vary a lot more with the smaller sample.



ANSWERS

1. The correct graph is E. Many students will initially choose A.
2. Students will likely come up with one of these lists:

				3, 3	3, 4	4, 4				
		2, 2	2, 3	2, 4	2, 5	3, 5	4, 5	5, 5		
1, 1	1, 2	1, 3	1, 4	1, 5	1, 6	2, 6	3, 6	4, 6	5, 6	6, 6

					6, 1					
				5, 1	5, 2	6, 2				
			4, 1	4, 2	4, 3	5, 3	6, 3			
		3, 1	3, 2	3, 3	3, 4	4, 4	5, 4	6, 4		
	2, 1	2, 2	2, 3	2, 4	2, 5	3, 5	4, 5	5, 5	6, 5	
1, 1	1, 2	1, 3	1, 4	1, 5	1, 6	2, 6	3, 6	4, 6	5, 6	6, 6

3. The second list includes both orders for the two dice. For example, 3, 4 and 4, 3.

4. Some students will argue that the step model is better, saying that order doesn't matter; 2, 3 and 3, 2 are the same thing and shouldn't be counted twice. Some students may argue that the triangle model is best, but may not be able to explain why the order is important. In fact, this is not an easy argument to make for rolling two dice, because we do not ordinarily keep track of the individual identities of the two dice. Using dice of two different colors, or rolling one die at a time, helps to make the importance of order clear.
5. About the same number of 11s and 12s
6. More 11s than 12s
7. Sketches will vary but should be roughly triangular in shape with a peak over a sum of 7.
8. Sketches will vary but should be roughly triangular in shape.
9. The triangle model
10. With a sample size of 1000 rolls, students are almost guaranteed to get more sums of 11 than sums of twelve. With a sample size of 100, students will often get more sums of twelve than sums of eleven.
11. The triangle model
12.

Sum	2	3	4	5	6	7	8	9	10	11	12
Theoretical Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$
	0.027	0.056	0.083	0.111	0.138	0.167	0.138	0.111	0.083	0.056	0.027