OVERVIEW

The primary goal of this activity is for students to recognize and generalize patterns in types of counting numbers. Specifically, students will create different arrangements of the numbers from 1 to 100; color all of the square numbers, triangular numbers, or prime numbers; and then describe, explain, and extend the patterns they see.

A secondary goal is to show how you can use TinkerPlots to explore non-statistical concepts and data. This activity, in particular, uses data about classes of numbers to develop number sense and pattern recognition.

This activity is rather long and may take more than one class period to complete. Because the activity is divided into three explorations—squares, triangular numbers, and primes—you could break it up over three consecutive half-days. You could also divide the activity into three completely separate activities that you do independent of each other, or you could pick and choose from among them.

Activity Time: One or two class periods

Objectives

- Select, apply, and translate among mathematical representations.

- Describe classes of counting numbers according to characteristics (specifically, square numbers, triangular numbers, and prime numbers).

- Represent, describe, extend, and generalize geometric and numeric patterns.

Common Core Standards Addressed

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

Grade 4, Operations & Algebraic Thinking Standard 5

Prerequisites

- Students should be familiar with *square numbers, prime numbers,* and *triangular numbers.* These types of numbers are described on the worksheet, but having prior experience with these classes of numbers will be extremely helpful.

- It is best if students have first done the activity Factors. That activity will give students experience with the type of data contained in the TinkerPlots document and with making stacks of rectangular icons.

Materials

- Number Properties worksheet (one copy per student)

- **Number Properties.tp**

LESSON PLAN

Introduction (10 minutes)

Hand out the Number Properties worksheet.

Talk through the introduction, and be sure students understand this vocabulary.

Note that the worksheet uses *square number* and *perfect square* synonymously. This could confuse some students. Also, the collection includes attributes called *square* and *perfect_square*. For this activity, students need to use *perfect_square*, which tells whether the number is a perfect square; *square* is a numeric attribute that gives the value when you square the number.

Plot and Investigate (35 minutes)

Have students move to computers and open the file **Number Properties.tp**. Support them as they work through the remainder of the worksheet

Wrap-Up (10–30 minutes)

The Answers section at the end of this document includes a lot of detail about the variety of approaches and solutions students might find. If you have enough time available, you could likely spend up to 30 minutes having students share their discoveries.

Extensions (optional)

1. Students will likely notice that there is an attribute called *perfect_cube*. You might define *cubic numbers* and have them look for patterns in the perfect cubes. Because there are only four perfect cubes less than 100, students might have a difficult time detecting any patterns.

    You may want to have students add additional cases, say for 101 to 250 (or higher), in order to see more of a pattern. *Note:* Most of the attributes in **Number Properties.tp** are calculated with formulas, so it will be very easy for students to add more cases. (Choose **New Cases** from the **Data** menu and enter the desired number of new cases. Values for all the attributes except *prime* will be automatically computed.)

2. Students will also likely notice that there are attributes called *square, square root,* and *cube*. These numeric attributes perform the indicated function on *number*. For students in grades 6–8 who are beginning to learn about function plots, you can create graphs of these attributes by fully separating *number* on the horizontal axis and fully separating *square,* for example, on the vertical axis. Students can study the general shape of these function curves and describe similarities and differences.

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3. You may be familiar with the Sieve of Eratosthenes. Eratosthenes (276–194 B.C.) was a Greek mathematician who devised a way to "strain out" prime numbers from composites. Essentially, because prime numbers cannot be a multiple of another number, you cross out all numbers that you know are multiples of 2, 3, 4, 5, and so on. You could do the Sieve with TinkerPlots, but it is better to do it with an old-fashioned number grid on a sheet of paper. Draw a number grid, or print a stacked plot, and cross off all the multiples of 2, then all the multiples of 3, and so on, up through the multiples of 17. Students will notice that for several of the factors no additional numbers get crossed out—their factors have already been eliminated. For example, when you get to 10 you find that all its multiples have already been eliminated by its factors 2 and 5.

ANSWERS

4. Answers will vary depending on grade level and pattern-recognition skills.

    In the simplest case, students should notice that the perfect squares are always in the first or fourth column. They might further notice that the perfect squares in the first column are always odd and that those in the fourth column are always even.

    Some students may notice patterns in the number of cases between the perfect squares. For example, in the first column the second perfect square is 2 rows above the first, the third perfect square is 4 rows above the second, the fourth perfect square is 6 rows above the third, and so on. Furthermore, in the fourth column the second perfect square is 3 rows above the first, the third perfect square is 5 rows above the second, the fourth perfect square is 7 rows above the third, and so on. (*Note:* Some students may describe the separation by the number of rows *between, but not including*, the perfect squares, in which case they might describe the first column's pattern as "1 row between..., 3 rows between..., etc.")

    Some students may notice that *consecutive* perfect squares increase by odds: "4 is 3 more than 1, 9 is 5 more than 4, 16 is 7 more than 9, . . ." However, because this insight requires reading across the rows rather than focusing on the columns of highlighted numbers, it may be a rare response.

    Note: Some students might prefer to swap the axes in order to make the plot fit the window better. This is okay, but then the patterns in the numbers go from left to right.

5. The first perfect square after 100 will be in the first column, 10 rows above 81. The second will be in the fourth column, 11 rows above 100. The next will be in the first column, 12 rows higher. The next will be in the fourth column, 13 rows higher. And the fifth will be in the first column, 14 rows higher. (*Note:* Accept any answer that gets at this insight, including sketches that attempt to extend the rectangular stack.)

Some students may recognize that the numbers in each column increase by 4 as you go up, so they might calculate the perfect squares as

$81 + (4 \times 10) = 121$

$100 + (4 \times 11) = 144$

$121 + (4 \times 12) = 169$

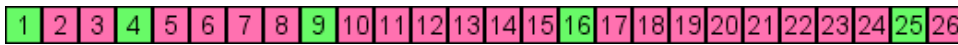$144 + (4 \times 13) = 196$

$169 + (4 \times 14) = 225$

Other students might sketch an extension of the diagram and fill in or count spaces to find the numbers.

Be aware that some students may simply calculate 11×11, 12×12, and so on. Although this is an effective way to find consecutive perfect squares, it does not use the pattern illustrated by the plot.

7. Answers will vary, although there are two ideal answers.

First, a stack that is 2 wide shows a pattern of perfect squares alternating from side to side. If you look at the distance between perfect squares in each column, the pattern is similar to the stack that was 4 wide, except the evens and odds are not consecutive. For example in the first column, you go up 4 rows, then 8 rows, then 12 rows. Other students might notice a pattern based on alternating back and forth: "go up 1 row and right, go up 3 rows and left, go up 3 rows and right, go up 5 rows and left, go up 5 rows and right, . . ."

Second, a stack that is 1 wide (or a continuous string) shows a pattern of increasing by odds: "up 3, up 5, up 7, up 9, . . ."

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 |

Some students might start experimenting with types of plots other than stacks of rectangular icons. For example, a fused circular graph shows the increasing-by-odds pattern, too.

35	36
33	34
31	32
29	30
27	28
25	26
23	24
21	22
19	20
17	18
15	16
13	14
11	12
9	10
7	8
5	6
3	4
1	2

9. Plots and answers will vary. The next five triangular numbers are 15, 21, 28, 36, and 45. A good answer is that triangular numbers increase by consecutive whole numbers: "go up 2, go up 3, go up 4, go up 5, . . ." Both the single stack and fused circular graph below show this pattern. (*Note:* If students count the case icons *between* triangular numbers, the pattern starts at 1.) To help students see this pattern, you might have them go back to the geometric pattern on the worksheet and construct the next numbers in the sequence. They should see that you make the next triangle by adding a bottom row one unit longer than the previous bottom row.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
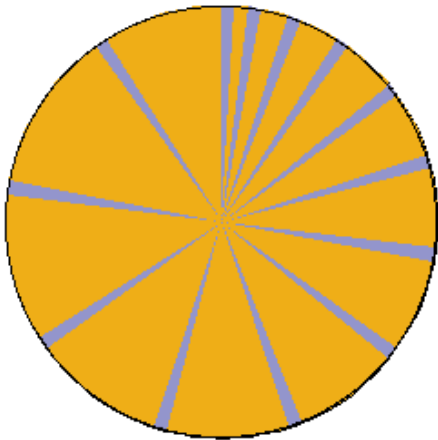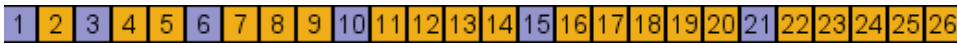
Also, triangular numbers always come in pairs of odds and evens: 1, 3 (two odds), 6, 10 (two evens), 15, 21 (two odds), 28, 36 (two evens), and so on. One way to highlight this pattern is to separate the icons by *triangular* and then color them by *even*. Again, you could ask students to go back to the triangular diagrams and challenge them to explain why this happens.
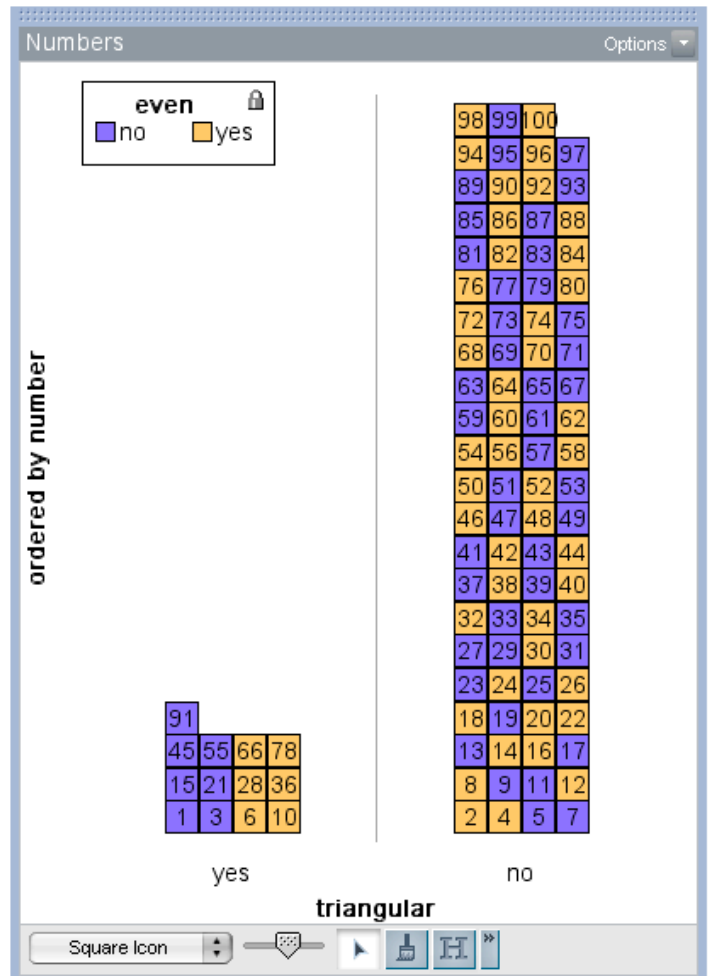
12. Plots and answers will vary. Unlike the perfect squares and triangular numbers, the primes have *no* consistent patterns. Some plots will appear to show patterns, but these patterns can't be extended for all primes. There are some things students can say, however, about primes, such as the fact that other than the number 2, they are all odd numbers.

Two interesting stacks of square icons are 5 wide and 6 wide, as shown below. The 5-wide stack shows that the primes tend to fall on certain diagonals. The 6-wide stack shows that the primes tend to be in the first and fifth rows. (Students could investigate why this is.)

96	97	98	99	100
91	92	93	94	95
86	87	88	89	90
81	82	83	84	85
76	77	78	79	80
71	72	73	74	75
66	67	68	69	70
61	62	63	64	65
56	57	58	59	60
51	52	53	54	55
46	47	48	49	50
41	42	43	44	45
36	37	38	39	40
31	32	33	34	35
26	27	28	29	30
21	22	23	24	25
16	17	18	19	20
11	12	13	14	15
6	7	8	9	10
1	2	3	4	5

97	98	99	100		
91	92	93	94	95	96
85	86	87	88	89	90
79	80	81	82	83	84
73	74	75	76	77	78
67	68	69	70	71	72
61	62	63	64	65	66
55	56	57	58	59	60
49	50	51	52	53	54
43	44	45	46	47	48
37	38	39	40	41	42
31	32	33	34	35	36
25	26	27	28	29	30
19	20	21	22	23	24
13	14	15	16	17	18
7	8	9	10	11	12
1	2	3	4	5	6

From having used fused circular plots for triangular numbers, some students might do the same for the primes. They might notice a slight increase in the distance between primes as the numbers get closer to 100.