OVERVIEW

Students explore the distribution of the number of boys in families with four children. First, students predict what the distribution of the number of boys in a family will look like. Then, they make a graph using index cards with the data of four-child families. Using TinkerPlots, they analyze data from 160 families with four children. Then, students systematically determine all possible simple outcomes and use them to construct the expected distribution of the number of boys in a family with four children, and calculate the probabilities of different outcomes. Finally, each student creates a sampler that builds four-child families and compares the results from this simulation to the expected distribution.

Activity Time: Two to three class periods

Objectives

- Use data to test expectations about the results of a chance experiment.
- Generate all possible simple outcomes to create a sample space and calculate the probability of various events.
- Understand the effect of sample size on the variability of actual results.

Common Core Standards Addressed

Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

Grade 7, Statistics and Probability Standard 1

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

Grade 7, Statistics and Probability Standard 2

Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of discrepancy.

Grade 7, Statistics and Probability Standard 7

Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

Grade 7, Statistics and Probability Standard 7b

Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

Grade 7, Statistics and Probability Standard 8a

Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

Grade 7, Statistics and Probability Standard 8b

Design and use a simulation to generate frequencies for compound events.

Grade 7, Statistics and Probability Standard 8c

Prerequisites

- Recognize that small samples do not always produce the expected results (perhaps by doing the activity "Wink, Blink, and Stare").

Materials

- Index cards of four-child families
- Blank index cards for students to add four-child families
- Markers (one per student)
- Paper axis labeled from 0 to 4 (or use index cards)
- Blank "four bars" (1-by-4-grid rectangles cut out of large-grid graph paper)
- Stickers in two colors (one color labeled G, the other B)
- Four-Child Families worksheet (one per student, perhaps page 1 and page 2 handed out separately)
- **Four-Child Families.tp**

LESSON PLAN

DAY 1

Before Class

Prepare a set of about 30 index cards, writing the genders in birth order of the children of families with four children. For example, "G, B, B, B." Include the family name on each card. The activity is most effective if you collect your own data from family and friends who have, or know, families with four children. You can also use the data at the end of these notes. It is important that students view these as real data, which is why it is important to include the family name (or another identifier) on each card.

Find an area of the floor of your classroom that is visible to the whole class, and tape down a paper axis labeled from 0 to 4 (as in the photograph below) or use numbered index cards for axis values.

Introduction (5 minutes)

Pose this question to the class:

> *A couple plans on having two children, and they hope they'll end up with one boy and one girl. What is that probability that this will happen?*

Give students some time to work through this on paper. Have volunteers give and explain their answers. However, it's not important to come to agreement at this point. Then tell them that they will be looking at families with four children.

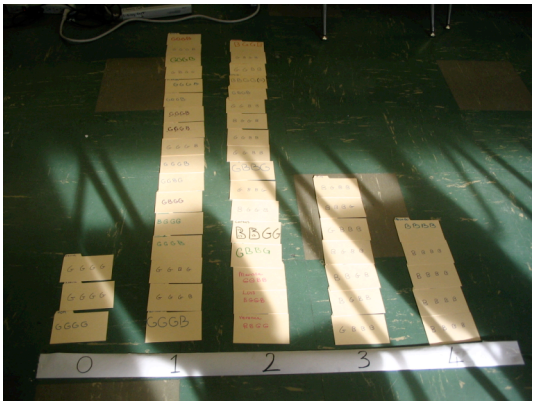
Card Sorting (20 minutes)

Hand out the worksheet. (You might hand out only page 1 today, as you'll likely do page 2 tomorrow.) Show the 30 prepared index cards, which already have families on them. Talk about some of the families, especially if there are some you know personally, so that students understand that the cards represent real families. Give students blank index cards to add the gender sequences (and family names) of any more four-child families that they may know.

Collect all the cards and explain that students are going to arrange them as a class according to the number of boys in each family. Agree as a class that there can be zero, one, two, three, or four boys in a family with four children, and point out the paper axis (or index cards with the numbers on them).

Place six random cards above the paper axis in the appropriate spot and say:

> *We'll continue to place all the cards, and we'll end up with a plot. Which of the three plots on the worksheet do you expect the plot of cards to look most like?*



Have each student make a decision and mark it on his or her worksheet. On the board, record the number of students who select each plot. Ask for rationales from different students, starting with anyone who selected Plot A, then Plot B, then Plot C. Let them give their reasons, but don't tell them which one is correct.

Build the card plot where all students can see it. You might ask some students to help you place a few of the cards. The picture here shows an example of what you might end up with.

After building the plot, have a brief discussion about what the real plot shows and which plot on the worksheet it most resembles. Ask:

> *Which of the three plots on your worksheet does this plot look like?*
>
> *Why do you think the plot has this shape?*

Some students may make an argument based on the number of ways to get each outcome. In particular, they might now see that all boys or all girls doesn't occur as often as other simple outcomes because there is only one way to get them. However, with this small of a sample, don't be surprised if it is not possible to decide between the three graphs based only on this data. The picture shown appears to suggest that it is more likely for a family with four children to have one boy than it is to have two. If students have done the activity "Wink, Blink, and Stare," remind them that they learned that small samples often gave unusual

results, so we should look at a larger sample. Otherwise mention another context in which they have encountered this concept.

Looking at 160 Families (15 minutes)

Tell students that you have the data for 160 families. Have students go to computers, or project TinkerPlots, and open **Four-Child Families.tp.** To build the distribution, students will need to drag the attribute *Count_boys* to the plot, separate the data, and stack them. (If students don't know how to do this, you should demonstrate. See the TinkerPlots movie "TinkerPlots Basics.")

Wrap-Up (5 minutes)

Ask students which of the three graphs on their worksheet the data from 160 families looks most like and whether they can explain why it would be that shape. Some students will suggest that the number of ways you can make families of various types has something to do with it. Tell students they will explore why the distribution has this shape tomorrow.

DAY 2

Constructing All Outcomes (15 minutes)

Show the cards from the previous day. Read a few aloud, and write them on the board. Then say:

> *Each of these families is a **simple outcome.** Can you name some more simple outcomes? If we write down every possible simple outcome, how many will there be?*

If you haven't already done so, hand out page two of the Four-Child Families worksheet. Tell students that they will investigate this question today by completing an activity. Write down one specific sequence on the board, such as "G, B, G, G." Explain the directions:

> *In the centers of your tables are piles of blank bars, like this one. I'll hand out sheets of green and blue stickers. The green stickers are labeled "G" for "girl" and the blue stickers "B" for "boy." Make a bar for each different four-child family order.*

Point to one of the families and make an example bar corresponding to that outcome.

Have students work in pairs or small groups to make as many different bars (families) as they can. They need sufficient space to arrange the bars they make. If they have individual desks, they could push two of them together. Give them enough time to do this and check their work, but don't tell them how many there are (16) or how they should do this.

Discussion (15 minutes)

After they have completed the task, ask:

How many different bars did you make? How do you know you have made all the possible bars?

As part of the discussion, ask students what made this question challenging. Two issues that should come up are:

- How to make sure there are no repeats
- How to know when you have all outcomes

Some groups are likely to have organized the bars similar to the display shown on the previous page, in which bars are grouped by the number of boys in the family. You might have students walk around as a group to look at other groups' work. Later, you may want to post one of these student sets somewhere in the classroom for reference.

Point out the different ways students have arranged the bars to help them keep track of which ones they had already made. Pick one that is organized by the number of boys, and call attention to the shape of this arrangement. Ask students:

Which of the three graphs does your arrangement look most like?

Does the arrangement here explain the shape of the distribution?

Suppose a couple is planning to have 4 children and they want two boys and two girls. They want to know what the probability is that this will happen. What is your answer?

Once students answer these questions, emphasize the importance of counting the number of ways something can happen.

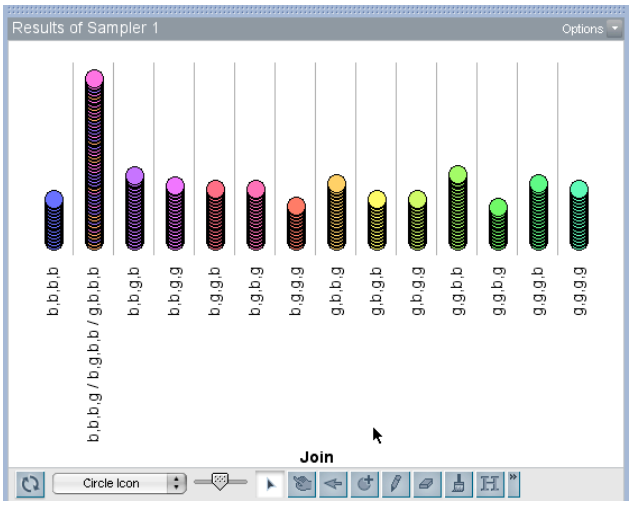
Calculating Probabilities (15 minutes)

Have students answer the question in Step 9 on the worksheet. You may want to go over the more challenging ones as a class after having students try them on their own.

DAY 3

Building a TinkerPlots Model (30 minutes)

To collect even more data, have students use TinkerPlots to build a model to simulate four-child families. If your students have prior experience building models in TinkerPlots, (such as the activity "Building a Data Factory") you can have students do this on their own. If not, you may wish to do this together as a class. (See the TinkerPlots movie "Building a Data Factory".) Students may need help figuring out how to make a plot that combines simple outcomes with the same number of boys. Two ways to do this are described in the next paragraphs.



One way to make a plot that combines simple outcomes is to drag simple icons with the same number of boys into the same bin. In this plot, the second bin from the left contains three of the four simple outcomes for families with 3 boys. The fourth simple outcome for this event is in the bin to its right. To move this group into the bin other four child families with three boys, click an icon in the bin and drag it into the bin to its left.

Another way to make this type of plot is to make a new attribute in the results table and use a formula. Go to the **Options** menu in the results table and choose **Results Options**. This opens an inspector panel with attributes you can add to the table. Click the box next to "Count '?' in Joined Values" and enter "b" in the dialog box. This creates an attribute, "Count_b," which records the number of boys in each simulated family.

After students have answered the questions in Steps 10 and 11, bring them back together and have them share their answers.

> *Did the shape you got for the distribution of number of boys in four-child families look like the shape you "expected" based on the sample space?*

Together as a class, change Repeat to 2000 and run the sampler. Draw several samples of this size. At this size, the distribution of the number of boys is four-child families will look almost exactly like the expected distribution. That is, the bars for zero and four boys will be the shortest and nearly the same height, the bars for one and three boys will be the next tallest and nearly the same height, and the bar for 2 boys will be the tallest. Remind students that they have made similar observations with experiments in past activities (perhaps the activity "Wink, Blink, and Stare").

Now change the sampler to take a sample the same size as the one you used to make the plot with index cards. Click the axis end values and change them so the minimum value is 0 and the maximum value is four, so that the axis will not change from sample to sample.

Ask students what they expect to observe when you run the sampler, and why. Some students will believe that it will still look like the distribution we expect. Others may anticipate that the results are going to sometimes look quite different.

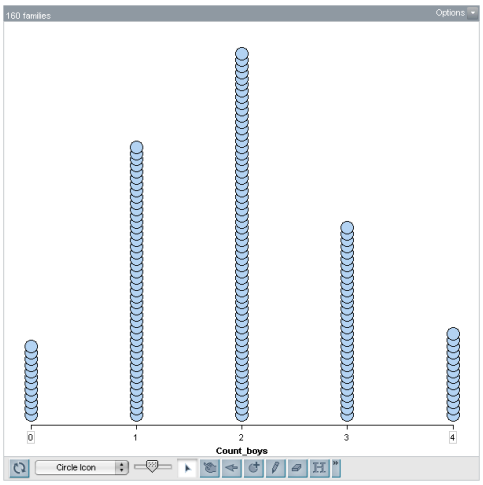
After students have answered, run the sampler and take several samples, pausing each time so students can observe the distribution. For a sample of 50, it is not unusual for the highest bar to correspond to a family with one or 3 boys. Each graph will look different, often with no families that have zero or four boys. Summarize what students have learned by saying that with small samples, it is easy to get results by chance that do not match our expectations.

Wrap-Up (10 minutes)

Ask students to share their answers to Step 11 with the class. Students should come away with two main understandings:

- To determine probabilities, first determine how many ways something can turn out.
- Very big samples often give results very close to what we expect, but small samples often give results that are different from what we expect.

ANSWERS

1. Answers will vary. Not many students will choose the correct answer, Plot C, and those who do are often unable to explain why. Most students will choose Plot B, arguing that it would be difficult to have either all boys or all girls, but everything else is equal. Some students will choose Plot A arguing that "this is chance, so everything is equal." Students will use data and an analysis of sample space to gain an understanding of why Plot C is correct, so don't share the correct answer too early.
2. Answers will vary.
3. Student sketches should reflect the general shape of the results in their sample. Don't be surprised if the sketches don't look like the theoretical distribution. This happens because 40 or 50 is too small a sample to reliably see the shape of the theoretical distribution. For example, see the picture on page 2.
5. 

7. 16

8.

		B, B, G, G		
		B, G, B, G		
	B, G, G, G	B, G, G, B	B, B, B, G	
	G, B, G, G	G, B, B, G	B, B, G, B	
	G, G, B, G	G, B, G, B	B, G, B, B	
G, G, G, G	G, G, G, B	G, G, B, B	G, B, B, B	B, B, B, B
0 boys	1 boy	2 boys	3 boys	4 boys

9. a. $\frac{6}{16} = \frac{3}{8} = 37.5\%$ b. $\frac{1}{16} = 6.25\%$ c. $\frac{1}{16} = 6.25\%$ d. $\frac{4}{16} = \frac{1}{4} = 25\%$
   e. $\frac{4}{16} = \frac{1}{4} = 25\%$ f. $\frac{1}{16} = 6.25\%$ g. $\frac{11}{16} = 68.75\%$ h. $\frac{5}{16} = 31.25\%$

11. Answers will vary. Ideally, students will realize that to determine probabilities, you should first determine how may ways the event you are interested in can happen; and that very big samples often give a distribution close to what we expect, but very small samples often differ from what we expect.

Additional Data for Cards

Name	Children's gender
Robinson	G B G G
Leach	G G G B
Uskova	G G B G
Bublik	B B B B
Baris	G B B B
D'Ambrosio	B B B B
Malachowski	G B B G
Chu	B B B B
Khalil	G G G G
Wilson J.	G B G G
Fullers	B G G G
Pinkney	G G G B
Scott	G G G B
Wilson G	G G G B
Westbrook	B G G B
Aunt A	G G B G
Uncle S	G B G B
Crliz	B B G G
Aunt	G G G B
Skwira K	G G G B
Cortes	B B G G
Mom	G G G G
Rodricjckz	G B B G

Name	**Children's gender**
Aunt Poortare	G G G B
Pinkey	G G G B
Veronica	B B G G
Mercado	B B B B
Luis	B G G B
Marissa	G G B B
Sandra	G B B G
Kreetong	B B G G